

STEP II, 1999, Q11

- 11 An automated mobile dummy target for gunnery practice is moving anti-clockwise around the circumference of a large circle of radius R in a horizontal plane at a constant angular speed ω . A shell is fired from O , the centre of this circle, with initial speed V and angle of elevation α . Show that if $V^2 < gR$, then no matter what the value of α , or what vertical plane the shell is fired in, the shell cannot hit the target.

Assume now that $V^2 > gR$ and that the shell hits the target, and let β be the angle through which the target rotates between the time at which the shell is fired and the time of impact. Show that β satisfies the equation

$$g^2\beta^4 - 4\omega^2V^2\beta^2 + 4R^2\omega^4 = 0.$$

Deduce that there are exactly two possible values of β .

Let β_1 and β_2 be the possible values of β and let P_1 and P_2 be the corresponding points of impact. By considering the quantities $(\beta_1^2 + \beta_2^2)$ and $\beta_1^2\beta_2^2$, or otherwise, show that the linear distance between P_1 and P_2 is

$$2R \sin\left(\frac{\omega}{g}\sqrt{V^2 - Rg}\right).$$



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