

STEP II, 1999, Q1

1 Let $x = 10^{100}$, $y = 10^x$, $z = 10^y$, and let

$$a_1 = x!, \quad a_2 = x^y, \quad a_3 = y^x, \quad a_4 = z^x, \quad a_5 = e^{xyz}, \quad a_6 = z^{1/y}, \quad a_7 = y^{z/x}.$$

- (i) Use Stirling's approximation $n! \approx \sqrt{2\pi} n^{n+\frac{1}{2}} e^{-n}$, which is valid for large n , to show that $\log_{10}(\log_{10} a_1) \approx 102$.
- (ii) Arrange the seven numbers a_1, \dots, a_7 in ascending order of magnitude, justifying your result.



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