

STEP II, 1998, Q7

7 Let

$$\begin{aligned}f(x) &= \tan x - x, \\g(x) &= 2 - 2 \cos x - x \sin x, \\h(x) &= 2x + x \cos 2x - \frac{3}{2} \sin 2x, \\F(x) &= \frac{x(\cos x)^{1/3}}{\sin x}.\end{aligned}$$

- (i) By considering $f(0)$ and $f'(x)$, show that $f(x) > 0$ for $0 < x < \pi/2$.
- (ii) Show similarly that $g(x) > 0$ for $0 < x < \pi/2$.
- (iii) Show that $h(x) > 0$ for $0 < x < \pi/4$, and hence that

$$x(\sin^2 x + 3 \cos^2 x) - 3 \sin x \cos x > 0$$

for $0 < x < \pi/4$.

- (iv) By considering $\frac{F'(x)}{F(x)}$, show that $F'(x) < 0$ for $0 < x < \pi/4$.



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