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AS Level Further Mathematics B (MEI)

Y414 Numerical Methods

Sample Question Paper Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- In each question you must show sufficient detail of the method(s) which you are using.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

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Answer **all** the questions.

1 The numbers x and y are approximated to three significant figures by $X = 4.15$ and $Y = 342$.

(i) Calculate the maximum possible relative error in each of these approximations. **[2]**

(ii) Hence calculate an estimate of the maximum possible relative error in using $\frac{Y}{X}$ as an approximation to $\frac{y}{x}$. **[2]**

2 The following table gives some values of a function $\mathrm{g}(x)$.

x	0	1	2	3	4
$\mathrm{g}(x)$	2	1	a	11	b

(i) Construct a difference table as far as the third differences. **[2]**

(ii) Given that $\mathrm{g}(x)$ is quadratic, find the values of a and b. **[3]**

(iii) Express $\mathrm{g}(x)$ in the form $px^2 + qx + r$, where p, q and r are numbers to be determined. **[3]**

3 It is required to solve the equation $\mathrm{f}(x) = 0$, where $\mathrm{f}(x) = x^5 - 3\sqrt{x} + 1$, by using an iterative method.

(i) Obtain a Newton-Raphson formula for successive iterations. **[3]**

With a starting value of $x_0 = 1$ the following iterates are generated by the Newton-Raphson formula for this equation.

1
1.285 714 286
1.195 628 311
1.177 204 057
1.176 497 074
1.176 496 062
1.176 496 062

(ii) (*A*) Calculate the ratios of differences for these iterates. **[2]**

(*B*) State what the values calculated in part **(ii)** (*A*) tell you about the speed of convergence of the Newton-Raphson iteration in this case. **[1]**

4 In a certain country the annual tax bill for each taxpayer is calculated to the nearest cent. When the data are transferred to the data base at the tax office the software chops each bill to the nearest dollar. There are 10 000 000 taxpayers, and 1 dollar = 100 cents.

(i) The amount which this chopping will lose the government in each tax year is modelled by assuming that, for each taxpayer, the maximum possible is lost. Use this model to calculate the amount which the government loses each tax year. **[2]**

(ii) Explain why the model in part **(i)** is unlikely to give a good estimate of the money lost by the government each tax year. **[1]**

(iii) Calculate the expected loss in one tax year. **[1]**

(iv) Explain the modelling assumption which underlies your calculation in part **(iii)**. **[1]**

5 A spreadsheet is used to calculate $\sum_{r=1}^{12} \frac{1}{r^9}$.

The following spreadsheet output shows the first steps in the process.

	A	B
1	*r*	f(*r*)
2	1	1
3	2	0.001953125
4	3	5.08053E-05
5	4	3.8147E-06
6	5	0.000000512
7	6	9.9229E-08
8	7	2.47809E-08
9	8	7.45058E-09
10	9	2.58117E-09
11	10	0.000000001
12	11	4.24098E-10
13	12	1.93807E-10
14		

Cell B3 contains the formula =1/(A3^9).

(i) State the purpose of this formula. **[1]**

(ii) Write down a formula which can be entered into cell B14 to complete the calculation. **[2]**

When the calculation is completed correctly, the value 1.002 008 393 is displayed in cell B14.

When $\sum_{r=1}^{20} \frac{1}{r^9}$ is calculated in a similar way, the same value, 1.002 008 393, is displayed in a cell.

(iii) Explain why this happens. **[2]**

6 **(i)** (*A*) Sketch the graphs of $y = 2^x$ and $y = 3x + 2$ for $-5 \le x \le 5$ on the same coordinate axes. **[2]**

(*B*) Hence state the number of real roots of the equation $2^x - 3x - 2 = 0$ for $-5 \le x \le 5$. **[1]**

(ii) Use the iteration $x_{r+1} = \dfrac{\left(2^{x_r} - 2\right)}{3}$ with $x_0 = 0$ to find the root near $x = 0$ correct to 5 decimal places. **[3]**

(iii) The iteration $x_{r+1} = \dfrac{\log_{10}\left(3x_r + 2\right)}{\log_{10} 2}$ may be used successfully to find the root near $x = 4$.

A student starts by entering `= 4` in cell A1 of a spreadsheet.

In cell A2 he enters `=LOG10(3A1)+2/LOG10(2)`.

This formula is incorrect. Write down the correct formula needed to obtain the result of the first iteration on the spreadsheet. **[2]**

7 When a 5 kg block of kryptonite is immersed in water it slowly dissolves. The mass of kryptonite remaining undissolved at time *t* hours, *A* kg, is modelled as follows.

- For the first 4 hours $A = 5 \times b^{\sqrt{t}}$ where *b* is a constant.
- Whatever the rate of dissolving is at *t* = 4, the kryptonite then dissolves at this constant rate until there is none left.

(i) After 4 hours the mass of kryptonite remaining is measured as 4.05 kg, correct to the nearest 50 grams. By making suitable calculations obtain the possible range of values for *b*. **[2]**

It is decided to apply the model using *b* = 0.9.

(ii) Use the forward difference method to

- produce an estimate of $\dfrac{dA}{dt}$ at *t* = 4 with *h* = 0.01,
- produce an estimate of $\dfrac{dA}{dt}$ at *t* = 4 with *h* = 0.001. **[3]**

(iii) Hence write down the best estimate of $\dfrac{dA}{dt}$ at *t* = 4 which can be justified from your answers in part **(ii)**. **[1]**

(iv) According to the model, when will all the kryptonite be completely dissolved? **[1]**

(v) The time when the kryptonite is completely dissolved is required to the nearest hour. Comment on whether the model will achieve this. **[1]**

8 The integral $I = \int_0^1 \sqrt{1-x^3}\,dx$ is to be evaluated numerically.

(i) (*A*) Find estimates of *I* by:

- using the midpoint rule with one strip;
- using the trapezium rule with one strip. **[4]**

(*B*) Hence find an estimate of *I* using Simpson's rule. **[2]**

Further approximations to *I* using the midpoint rule and the trapezium rule are found by successively reducing *h* by a factor of 2 as far as $h = 0.0078125$. These approximations are shown in the spreadsheet output below. Some values in the spreadsheet have been deliberately omitted.

	A	B	C	D
1	*h*	Midpoint	Trapezium	Simpson
2	1			
3	0.5	0.876251029	0.717707173	
4	0.25	0.854056168	0.796979101	
5	0.125	0.845891776	0.825517635	
6	0.0625	0.842943197	0.835704705	
7	0.03125	0.841889401	0.839323951	
8	0.015625	0.841514808	0.840606676	
9	0.0078125	0.841382011	0.841060742	

(ii) (*A*) Write down the best possible estimate of *I* which can be justified from these approximations. **[1]**

(*B*) Give a reason for the precision of your estimate in part **(ii)** (*A*). **[1]**

(iii) Give two different spreadsheet formulae which could be used to give the Simpson's rule estimate $S_{64} = 0.841\,212\,097$ in cell D8. The formulae should use cell references only from columns B and C. **[4]**

In order to evaluate *I* as accurately as possible, the column of Simpson's rule estimates was completed; the ratio of differences for these estimates was found to converge to approximately 0.353. This value was used to obtain an improved approximation by extrapolating to infinity. A second approximation was found by extrapolating with the theoretical value $r = \left(\frac{1}{2}\right)^4 = 0.0625$. The following results were obtained.

Extrapolation with $r = 0.353$	Extrapolation with $r = 0.0625$
0.841 309 2	0.841 279 1

(iv) State which of these values is more reliable and justify your answer. **[2]**

(v) Obtain the best possible estimate of *I*. You should explain your reasoning carefully. **[2]**

END OF QUESTION PAPER

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