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AS Level Further Mathematics B (MEI)

Y413 Modelling with Algorithms

Sample Question Paper Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **12** pages.

Answer **all** the questions

1 In Fig. 1 the weights on the arcs represent distances.

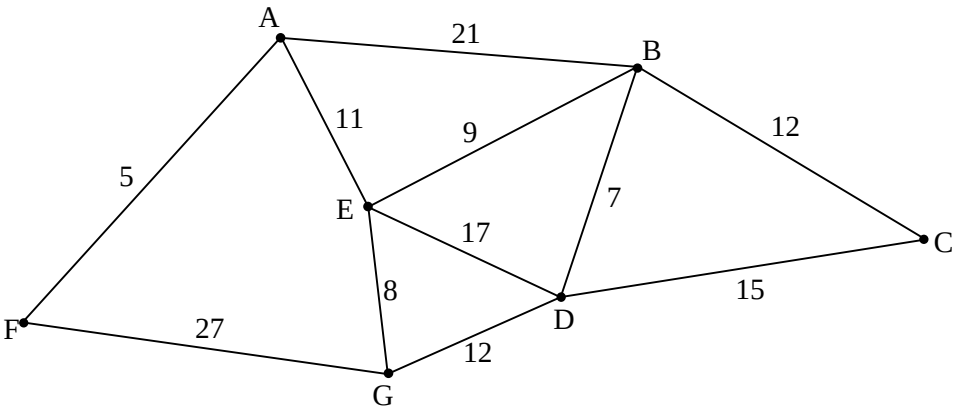

Fig. 1

Apply Dijkstra's algorithm to find the shortest path from A to D.

Give

- your shortest path

and

- its length. **[6]**

2 The instructions labelled Step 10 to Step 90 below describe a bubble sort algorithm to sort 4 numbers.

```
Step 10  Let i equal 1.
Step 20      Let j equal 1.
Step 30          If the jth number in the list is less than the (j+1)th, then swap them.
Step 40          Let the new value of j be j+1.
Step 50              If j is greater than 4–i, then go to Step 70.
Step 60              Go to Step 30.
Step 70      Let the new value of i be i+1.
Step 80  If i is equal to 4, then stop.
Step 90  Go to Step 20.
```

Four students take a test. Ali scores 57, Bill scores 67, Cleo scores 43 and Debbie scores 73.

(i) Use this bubble sort algorithm to rearrange the individuals from alphabetical order into descending order of their test scores. Record the names and scores in the order that they appear each time Step 70 is used. **[3]**

Ewan takes the test later, and his score of 60 is added to the list by comparing his score with the highest score, then with the second highest, and so on, until it can be put in the correct place.

(ii) Determine how many comparisons were made in using the bubble sort in part **(i)** and then inserting Ewan's score into the list. **[2]**

(iii) Describe how to amend the instructions so they give a bubble sort for 5 numbers. **[1]**

(iv) The five students are listed in alphabetical order. How many comparisons are made when your amended bubble sort is used to arrange their scores into descending order? You do not need to carry out the bubble sort. **[1]**

3 An industrial process is represented by the network shown in Fig. 3. The diagram also shows the precedences and durations, in minutes, for each activity, and the earliest event times for seven of the ten events.

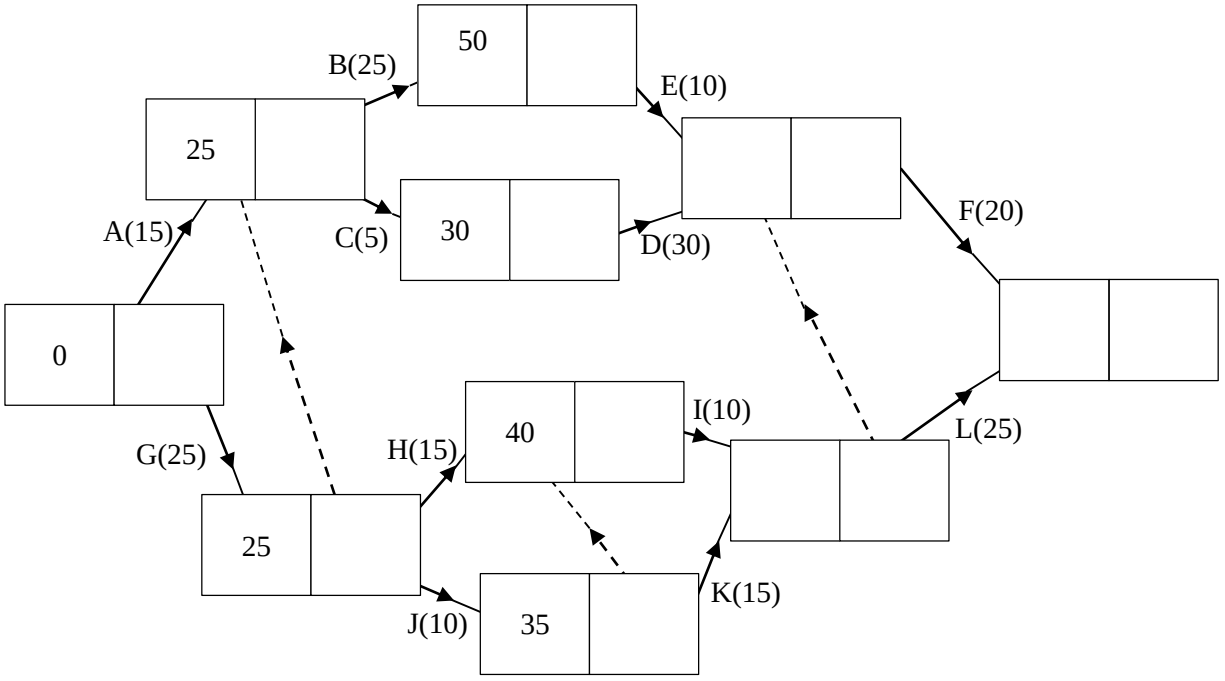

Fig. 3

(i) Find the minimum completion time for the process based on this information.

Identify

- the critical activities

and

- the total floats for the non-critical activities. **[5]**

Activities C and H need access to the same machine, so cannot happen simultaneously. Activities E and I need to be undertaken by the same person, so cannot happen simultaneously.

(ii) Explain why the process cannot be completed in the minimum completion time you found in part **(i).** **[3]**

(iii) Give a minimum completion time that takes these constraints into account. **[1]**

4 The table and the network in Fig. 4 represent the layout of cables joining nine junction boxes in a town; the weights on the arcs and the values in the table are the lengths of the cables, in kilometres.

	A	B	C	D	E	F	G	H	I
A		4	7		2				
B	4		4	5					
C	7	4		3	6				
D		5	3						
E	2		6			5			
F					5		3	1	
G						3		2	3
H						1	2		2
I							3	2	

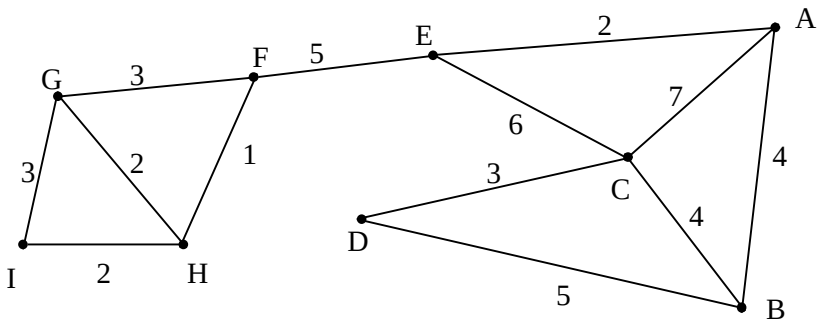

Fig. 4

Each month it costs £8 per kilometre to maintain the cables. Some of the cables are to be removed to save maintenance costs. The network must remain connected.

(i) (*A*) Which cables should be removed to save the maximum amount of money per month on maintenance? You should show sufficient working to make your reasoning clear. **[5]**

(*B*) What is the maximum amount of money which can be saved per month on maintenance? **[1]**

The cost of maintenance needs to be cut further. It is proposed to lay a new cable connecting the junction boxes at D and H. This cable costs £200 to lay, is 2 km long, and has the same maintenance costs as the other cables. When the new cable is laid, one or more of the old cables will be removed to obtain a further reduction in maintenance costs.

(ii) How many months will it take before the further reduction in maintenance costs is greater than the amount spent on laying the new cable? You should show sufficient working to support your reasoning. **[4]**

5 The following LP problem is to be solved.

$$\begin{aligned} \text{Maximise} \quad & P = \frac{1}{3}x + \frac{1}{2}y \\ \text{subject to} \quad & x + 2y \leq 9 \\ & 2x + 3y \leq 14 \\ & 2x + y \leq 10 \\ & x \geq 0 \\ & y \geq 0 \end{aligned}$$

The graph in Fig. 5 shows the feasible region for the problem.

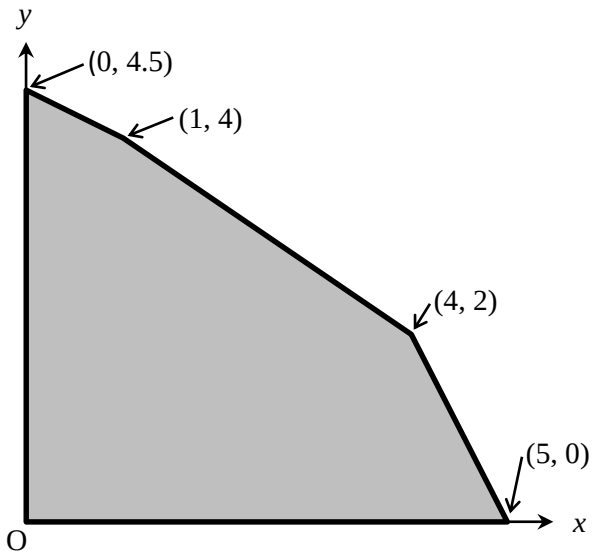

Fig. 5

(i) Use the graph to solve the LP problem. **[3]**

Chetan solves the problem using the simplex algorithm. His final tableau is shown below.

P	x	y	s_1	s_2	s_3	RHS
1	0	0	0	$\frac{1}{6}$	0	$\frac{7}{3}$
0	0	0	1	$-\frac{3}{4}$	$\frac{1}{4}$	1
0	1	0	0	$-\frac{1}{4}$	$\frac{3}{4}$	4
0	0	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	2

(ii) Interpret this tableau. **[3]**

(iii) (*A*) Perform another iteration using an entry in the s_3 column as the pivot element. **[2]**

(*B*) Comment on the result. **[1]**

6 Virginia is setting up an airline. She has a capital budget of \$500 million (\$500m) to buy aeroplanes. The capital budget cannot be used to pay running costs.

Large aeroplanes cost \$18m each and have a capacity of 250 passengers. Large aeroplanes have fixed costs of \$3m each per year and variable costs of \$2.90 per mile.

Medium aeroplanes cost \$15m each and have a capacity of 200 passengers. Medium aeroplanes have fixed costs of \$3m each per year and variable costs of \$2 per mile.

Small aeroplanes cost \$12m each and have a capacity of 150 passengers. Small aeroplanes have fixed costs of \$1.5m each per year and variable costs of \$2 per mile.

Virginia's company will fly transatlantic routes and domestic routes. The average distances and demands for these routes are as shown in the table below.

	Distance (miles)	Demand (passengers per year)
Transatlantic	5000	1 000 000
Domestic	1000	2 250 000

Each aeroplane will fly for 300 days a year. Each aeroplane can make two transatlantic flights per day, or four domestic flights per day.

Let NL be the number of large aeroplanes, TL the number of transatlantic flights per year using large aeroplanes and DL the number of domestic flights per year using large aeroplanes. Use similar variables for medium and small aeroplanes.

(i) Explain why the annual running cost of using large aeroplanes is given by $3\,000\,000\ \text{NL} + 14\,500\ \text{TL} + 2900\ \text{DL}$. **[3]**

(ii) The inequality $0.5\ \text{TL} + 0.25\ \text{DL} \leq 300\ \text{NL}$ models the availability of large aeroplanes. What does each side of the inequality represent? **[2]**

(iii) Virginia wishes to minimise her annual running cost. Formulate an LP to find how many aeroplanes of each type Virginia should buy so that she can satisfy demand within her capital budget.

- For each type of aeroplane you will need an availability inequality.
- For each distance category you will need an inequality to ensure that there is sufficient capacity.
- You will need an inequality to ensure that the capital budget is not exceeded. **[6]**

The LP is run in a spreadsheet LP solver and the following output is obtained.

Result: Solver found an integer solution within tolerance. All Constraints are satisfied.

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$K$6	objective	0	123845400

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$5	NL	0	6	Integer
$C$5	TL	0	162	Integer
$D$5	DL	0	6876	Integer
$E$5	NM	0	8	Integer
$F$5	TM	0	4797	Integer
$G$5	DM	0	6	Integer
$H$5	NS	0	3	Integer
$I$5	TS	0	1	Integer
$J$5	DS	0	3532	Integer

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$K$10	availability 3	-16.5	$K$10<=$L$10	Not Binding	16.5
$K$11	capacity 1	1000050	$K$11>=$L$11	Not Binding	50
$K$12	capacity 2	2250000	$K$12>=$L$12	Binding	0
$K$13	capital	264	$K$13<=$L$13	Not Binding	236
$K$8	availability 1	0	$K$8<=$L$8	Binding	0
$K$9	availability 2	0	$K$9<=$L$9	Binding	0
$B$5:$J$5=Integer					

(iv) Interpret the output to advise Virginia how many aeroplanes of each type she should buy. **[2]**

(v) Give two criticisms of the LP model. **[2]**

Virginia uses the model and the output from the LP solver to set the price of tickets. She decides to set the cost per mile at a lower rate for transatlantic flights than for domestic flights. She wishes her income from tickets to be at least $25 million more than her annual running costs.

(vi) Showing your calculations, suggest what price she might charge for tickets on transatlantic and domestic flights. **[4]**

END OF QUESTION PAPER

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