OCR
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AS Level Further Mathematics B (MEI)

Y410 Core Pure

Sample Question Paper Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question or part question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 The complex number z_1 is $1+\mathrm{i}$ and the complex number z_2 has modulus 4 and argument $\frac{\pi}{3}$.

(i) Express z_2 in the form $a+b\mathrm{i}$, giving a and b in exact form. **[2]**

(ii) Express $\frac{z_2}{z_1}$ in the form $c+d\mathrm{i}$, giving c and d in exact form. **[2]**

2 **(i)** Describe fully the transformation represented by the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. **[2]**

(ii) A triangle of area 5 square units undergoes the transformation represented by the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

Explaining your reasoning, find the area of the image of the triangle following this transformation. **[2]**

3 **(i)** Write down, in complex form, the equation of the locus represented by the circle in the Argand diagram shown in Fig. 3. **[2]**

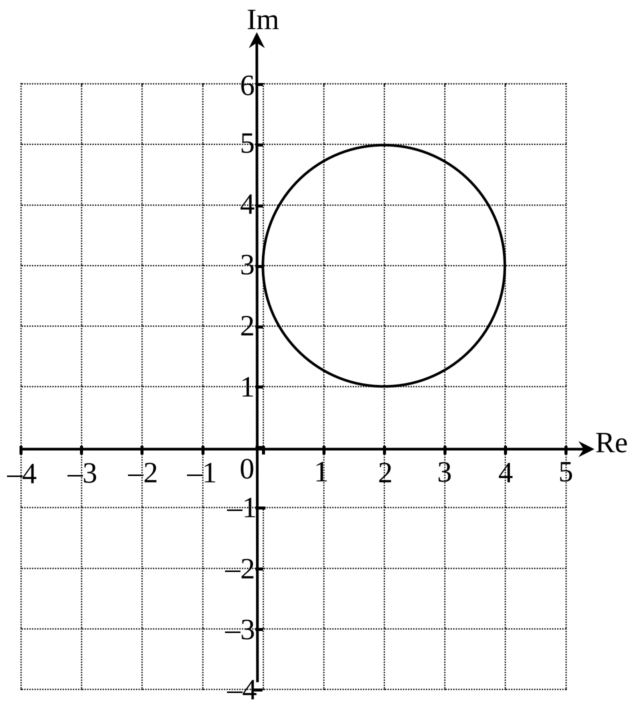

Fig. 3

(ii) On the copy of Fig. 3 in the Printed Answer Booklet mark with a cross any point(s) on the circle for which $\arg(z-2\mathrm{i})=\frac{\pi}{4}$. **[2]**

4 **(i)** Find the coordinates of the point where the following three planes intersect. Give your answers in terms of a.

$$\begin{aligned} x-2y-\ z&=\ 6\\ 3x+\ y+5z&=-4\\ -4x+2y-3z&=\ a \end{aligned}$$

[4]

(ii) Determine whether the intersection of the three planes could be on the z-axis. **[2]**

5 The cubic equation $x^3-4x^2+px+q=0$ has roots α, $\dfrac{2}{\alpha}$ and $\alpha+\dfrac{2}{\alpha}$.

Find

- the values of the roots of the equation,
- the value of p. **[7]**

6 **(i)** Show that, when $n=5$, $\sum_{r=n+1}^{2n} r^2 = 330$. **[1]**

(ii) Find, in terms of n, a fully factorised expression for $\sum_{r=n+1}^{2n} r^2$. **[4]**

7 The plane Π has equation $3x-5y+z=9$.

(i) Show that Π contains

- the point $(4,1,2)$

and

- the vector $\begin{pmatrix}1\\1\\2\end{pmatrix}$. **[4]**

(ii) Determine the equation of a plane which is perpendicular to Π and which passes through $(4,1,2)$. **[3]**

8 **In this question you must show detailed reasoning.**

(i) Explain why all cubic equations with real coefficients have at least one real root. **[2]**

(ii) Points representing the three roots of the equation $z^3 + 9z^2 + 27z + 35 = 0$ are plotted on an Argand diagram.

Find the exact area of the triangle which has these three points as its vertices. **[7]**

9 You are given that matrix $\mathbf{M} = \begin{pmatrix} -3 & 8 \\ -2 & 5 \end{pmatrix}$.

(i) Prove that, for all positive integers n, $\mathbf{M}^n = \begin{pmatrix} 1-4n & 8n \\ -2n & 1+4n \end{pmatrix}$. **[6]**

(ii) Determine the equation of the line of invariant points of the transformation represented by the matrix **M**. **[3]**

It is claimed that the answer to part **(ii)** is also a line of invariant points of the transformation represented by the matrix $\mathbf{M}^n$, for any positive integer n.

(iii) Explain *geometrically* why this claim is true. **[2]**

(iv) Verify *algebraically* that this claim is true. **[3]**

END OF QUESTION PAPER