

## AS Level Further Mathematics B (MEI)

### Y410 Core Pure

Sample Question Paper

Version 2

## Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

#### You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

#### You may use:

- a scientific or graphical calculator



### INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

### INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question or part question are shown in brackets [ ].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

- 1 The complex number  $z_1$  is  $1+i$  and the complex number  $z_2$  has modulus 4 and argument  $\frac{\pi}{3}$ .

(i) Express  $z_2$  in the form  $a+bi$ , giving  $a$  and  $b$  in exact form. [2]

(ii) Express  $\frac{z_2}{z_1}$  in the form  $c+di$ , giving  $c$  and  $d$  in exact form. [2]

- 2 (i) Describe fully the transformation represented by the matrix  $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ . [2]

(ii) A triangle of area 5 square units undergoes the transformation represented by the matrix  $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ .

Explaining your reasoning, find the area of the image of the triangle following this transformation. [2]

- 3 (i) Write down, in complex form, the equation of the locus represented by the circle in the Argand diagram shown in Fig. 3. [2]

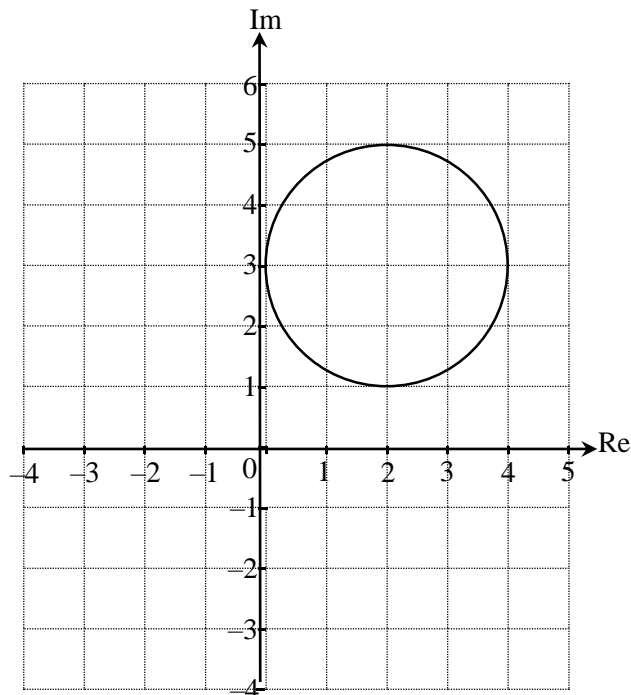


Fig. 3

- (ii) On the copy of Fig. 3 in the Printed Answer Booklet mark with a cross any point(s) on the circle for which  $\arg(z-2i) = \frac{\pi}{4}$ . [2]

## 3

- 4 (i) Find the coordinates of the point where the following three planes intersect. Give your answers in terms of  $a$ .

$$\begin{aligned}x - 2y - z &= 6 \\3x + y + 5z &= -4 \\-4x + 2y - 3z &= a\end{aligned}$$

[4]

- (ii) Determine whether the intersection of the three planes could be on the  $z$ -axis.

[2]

- 5 The cubic equation  $x^3 - 4x^2 + px + q = 0$  has roots  $\alpha$ ,  $\frac{2}{\alpha}$  and  $\alpha + \frac{2}{\alpha}$ .

Find

- the values of the roots of the equation,
- the value of  $p$ .

[7]

- 6 (i) Show that, when  $n = 5$ ,  $\sum_{r=n+1}^{2n} r^2 = 330$ .

[1]

- (ii) Find, in terms of  $n$ , a fully factorised expression for  $\sum_{r=n+1}^{2n} r^2$ .

[4]

- 7 The plane  $\Pi$  has equation  $3x - 5y + z = 9$ .

- (i) Show that  $\Pi$  contains

- the point  $(4, 1, 2)$

and

- the vector  $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ .

[4]

- (ii) Determine the equation of a plane which is perpendicular to  $\Pi$  and which passes through  $(4, 1, 2)$ . [3]

**8 In this question you must show detailed reasoning.**

(i) Explain why all cubic equations with real coefficients have at least one real root. [2]

(ii) Points representing the three roots of the equation  $z^3 + 9z^2 + 27z + 35 = 0$  are plotted on an Argand diagram.

Find the exact area of the triangle which has these three points as its vertices. [7]

9 You are given that matrix  $\mathbf{M} = \begin{pmatrix} -3 & 8 \\ -2 & 5 \end{pmatrix}$ .

(i) Prove that, for all positive integers  $n$ ,  $\mathbf{M}^n = \begin{pmatrix} 1-4n & 8n \\ -2n & 1+4n \end{pmatrix}$ . [6]

(ii) Determine the equation of the line of invariant points of the transformation represented by the matrix  $\mathbf{M}$ . [3]

It is claimed that the answer to part (ii) is also a line of invariant points of the transformation represented by the matrix  $\mathbf{M}^n$ , for any positive integer  $n$ .

(iii) Explain *geometrically* why this claim is true. [2]

(iv) Verify *algebraically* that this claim is true. [3]

**END OF QUESTION PAPER**

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