Oxford Cambridge and RSA

Accredited

A Level Further Mathematics B (MEI)

Y421 Mechanics Major

Sample Question Paper Version 2

Date – Morning/Afternoon

Time allowed: 2 hours 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **120**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

Section A (26 marks)

Answer **all** the questions

1 A particle P has position vector **r** m at time t s given by $\mathbf{r} = (t^3 - 3t^2)\,\mathbf{i} - (4t^2 + 1)\,\mathbf{j}$ for $t \geq 0$.
Find the magnitude of the acceleration of P when $t = 2$. **[4]**

2 A particle of mass 5 kg is moving with velocity $2\mathbf{i} + 5\mathbf{j}$ m s^{-1}. It receives an impulse of magnitude 15 N s in the direction $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$. Find the velocity of the particle immediately afterwards. **[3]**

3 The fixed points E and F are on the same horizontal level with EF = 1.6 m. A light string has natural length 0.7 m and modulus of elasticity 29.4 N. One end of the string is attached to E and the other end is attached to a particle of mass M kg. A second string, identical to the first, has one end attached to F and the other end attached to the particle. The system is in equilibrium in a vertical plane with each string stretched to a length of 1 m, as shown in Fig. 3.

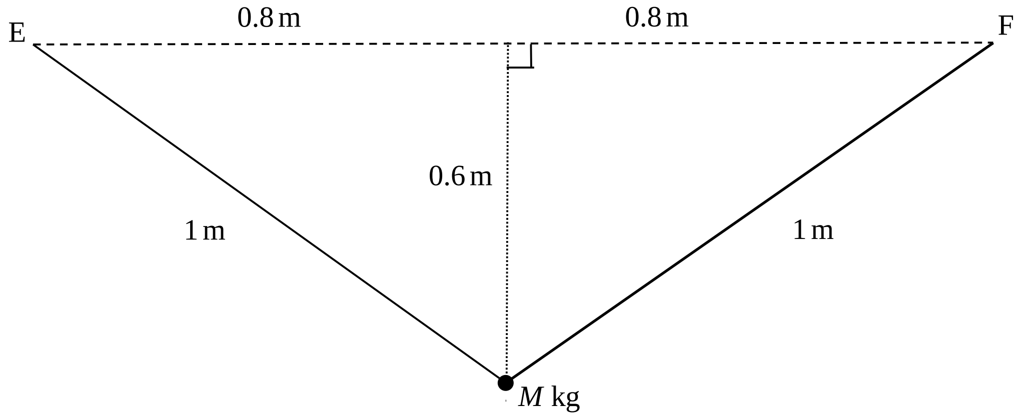

Fig. 3

(i) Find the tension in each string. **[2]**

(ii) Find M. **[3]**

4 A fixed smooth sphere has centre O and radius a. A particle P of mass m is placed at the highest point of the sphere and given an initial horizontal speed u.

For the first part of its motion, P remains in contact with the sphere and has speed v when OP makes an angle θ with the upward vertical. This is shown in Fig. 4.

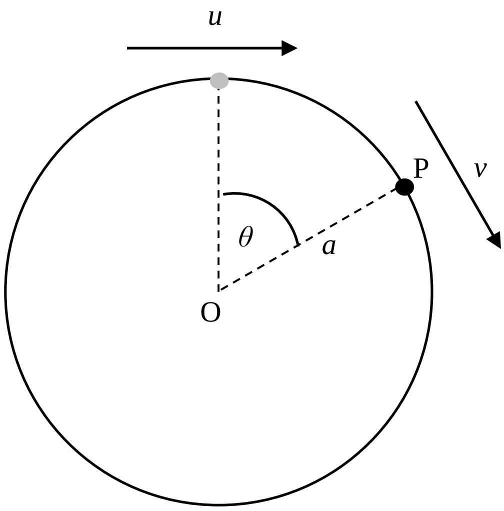

Fig. 4

(i) By considering the energy of P, show that $v^2 = u^2 + 2ga(1-\cos\theta)$. **[2]**

(ii) Show that the magnitude of the normal contact force between the sphere and particle P is $mg(3\cos\theta - 2) - \frac{mu^2}{a}$. **[2]**

The particle loses contact with the sphere when $\cos\theta = \frac{3}{4}$.

(iii) Find an expression for u in terms of a and g. **[2]**

5 Fig. 5 shows a light inextensible string of length 3.3 m passing through a small smooth ring R. The ends of the string are attached to fixed points A and B, where A is vertically above B. The ring R has mass 0.27 kg and is moving with constant speed in a horizontal circle of radius 1.2 m. The distances AR and BR are 2 m and 1.3 m respectively.

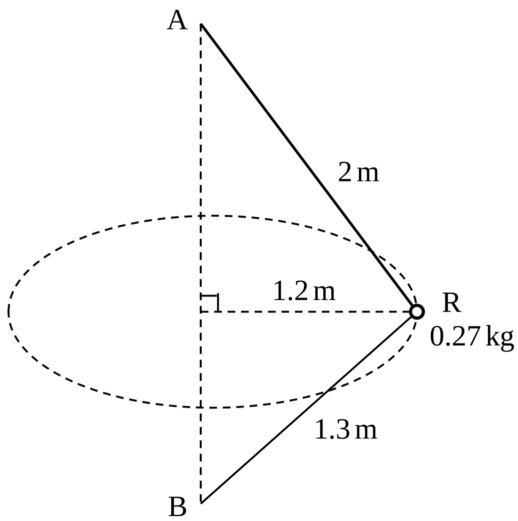

Fig. 5

(i) Show that the tension in the string is 6.37 N. **[4]**

(ii) Find the speed of R. **[4]**

Section B (94 marks)

Answer **all** the questions

6 Fig. 6 shows a pendulum which consists of a rod AB freely hinged at the end A with a weight at the end B. The pendulum is oscillating in a vertical plane. The total energy, E, of the pendulum is given by

$$E = \frac{1}{2}I\omega^2 - mgh\cos\theta \,,$$

where

- ω is its angular speed
- m is its mass
- h is the distance of its centre of mass from A
- θ is the angle the rod makes with the downward vertical
- g is the acceleration due to gravity
- I is a quantity known as the moment of inertia of the pendulum.

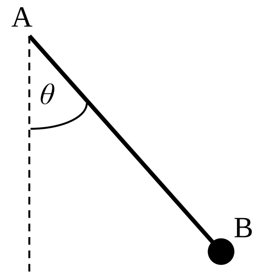

Fig. 6

(i) Use the expression for E to deduce the dimensions of I. **[4]**

It is suggested that the period of oscillation, T, of the pendulum is given by $T = kI^{\alpha}(mg)^{\beta}h^{\gamma}$, where k is a dimensionless constant.

(ii) Use dimensional analysis to find the values of α, β and γ. **[5]**

A class experiment finds that, when all other quantities are fixed, T is proportional to $\frac{1}{\sqrt{m}}$.

(iii) Determine whether this result is consistent with your answer to part **(ii)**. **[1]**

7 A uniform ladder of length 8 m and weight 180 N stands on a rough horizontal surface and rests against a smooth vertical wall. The ladder makes an angle of 20° with the wall. A woman of weight 720 N stands on the ladder. Fig. 7 shows this situation modelled with the woman's weight acting at a distance x m from the lower end of the ladder.

The system is in equilibrium.

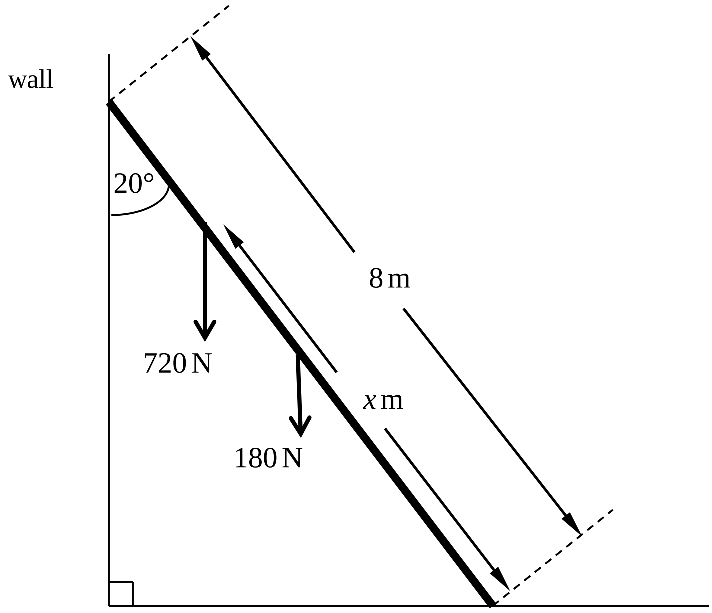

Fig. 7

(i) Show that the frictional force between the ladder and the horizontal surface is F N, where $F = 90(1+x)\tan 20^\circ$. **[4]**

(ii) (*A*) State with a reason whether F increases, stays constant or decreases as x increases. **[1]**

(*B*) Hence determine the set of values of the coefficient of friction between the ladder and the surface for which the woman can stand anywhere on the ladder without it slipping. **[4]**

8 A tractor has a mass of 6000 kg. When developing a power of 5 kW, the tractor is travelling at a steady speed of $2.5\,\text{m}\,\text{s}^{-1}$ across a horizontal field.

(i) Calculate the magnitude of the resistance to the motion of the tractor. **[2]**

The tractor comes to horizontal ground where the resistance to motion is different. The power developed by the tractor during the next 10 s has an average value of 8 kW. During this time, the tractor accelerates uniformly from $2.5\,\text{m}\,\text{s}^{-1}$ to $3\,\text{m}\,\text{s}^{-1}$.

(ii) (*A*) Show that the work done against the resistance to motion during the 10 s is 71 750 J. **[4]**

(*B*) Assuming that the resistance to motion is constant, calculate its value. **[3]**

The tractor can usually travel up a straight track inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{20}$, while accelerating uniformly from 3 m s^{-1} to 3.25 m s^{-1} over a distance of 100 m against a resistance to motion of constant magnitude of 2000 N.

The tractor develops a fault which limits its maximum power to 16kW.

(iii) Determine whether the tractor could now perform the same motion up the track.
[You should assume that the mass of the tractor and the resistance to motion remain the same.] **[7]**

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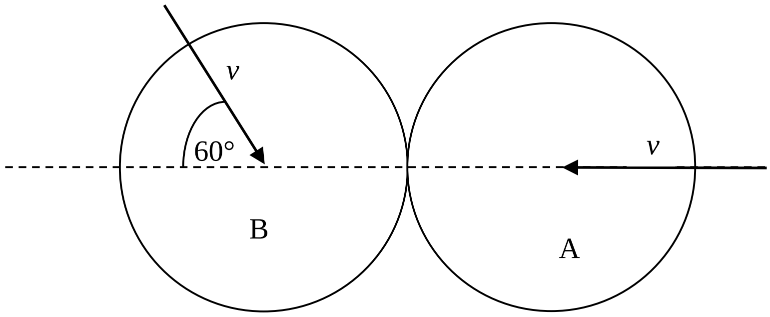

Fig. 9

Fig. 9 shows the instant of impact of two identical uniform smooth spheres, A and B, each with mass *m*. Immediately before they collide, the spheres are sliding towards each other on a smooth horizontal table in the directions shown in the diagram, each with speed *v*. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(i) Show that, immediately after the collision, the speed of A is $\frac{1}{8}v$. Find its direction of motion. **[6]**

(ii) Find the percentage of the original kinetic energy that is lost in the collision. **[7]**

(iii) State where in your answer to part **(i)** you have used the assumption that the contact between the spheres is smooth. **[1]**

10 In this question take $g = 10$.

A smooth ball of mass 0.1 kg is projected from a point on smooth horizontal ground with speed $65\,\text{m}\,\text{s}^{-1}$ at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. While it is in the air the ball is modelled as a particle moving freely under gravity. The ball bounces on the ground repeatedly. The coefficient of restitution for the first bounce is 0.4.

(i) Show that the ball leaves the ground after the first bounce with a horizontal speed of $52\,\text{m}\,\text{s}^{-1}$ and a vertical speed of $15.6\,\text{m}\,\text{s}^{-1}$. Explain your reasoning carefully. **[4]**

(ii) Calculate the magnitude of the impulse exerted on the ball by the ground at the first bounce. **[2]**

Each subsequent bounce is modelled by assuming that the coefficient of restitution is 0.4 and that the bounce takes no time. The ball is in the air for T_1 seconds between projection and bouncing the first time, T_2 seconds between the first and second bounces, and T_n seconds between the $(n-1)$th and nth bounces.

(iii) (*A*) Show that $T_1 = \frac{39}{5}$. **[2]**

(*B*) Find an expression for T_n in terms of n. **[2]**

(iv) According to the model, how far does the ball travel horizontally while it is still bouncing? **[3]**

(v) According to the model, what is the motion of the ball after it has stopped bouncing? **[1]**

11 The region bounded by the x-axis and the curve $y = \frac{1}{2}k\left(1 - x^2\right)$ for $-1 \leq x \leq 1$ is occupied by a uniform lamina, as shown in Fig. 11.1.

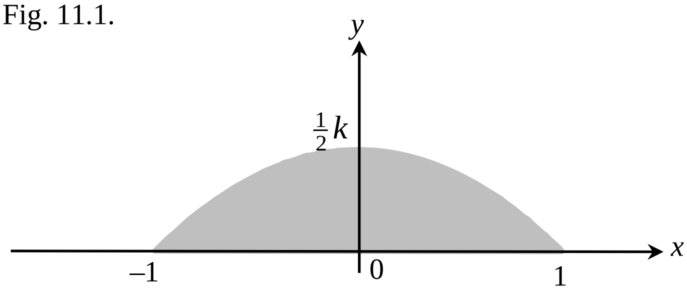

Fig. 11.1

(i) **In this question you must show detailed reasoning.**

Show that the centre of mass of the lamina is at $\left(0, \frac{1}{5}k\right)$. **[7]**

A shop sign is modelled as a uniform lamina in the form of the lamina in part **(i)** attached to a rectangle ABCD, where AB = 2 and BC = 1. The sign is suspended by two vertical wires attached at A and D, as shown in Fig. 11.2.

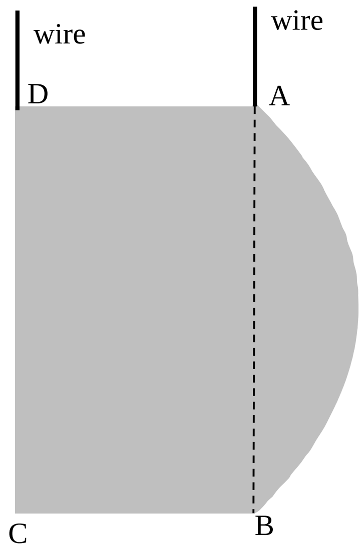

Fig. 11.2

(ii) Show that the centre of mass of the sign is at a distance

$$\frac{2k^2 + 10k + 15}{10k + 30}$$

from the midpoint of CD. **[4]**

The tension in the wire at A is twice the tension in the wire at D.

(iii) Find the value of k. **[5]**

12 Fig. 12 shows *x*- and *y*- coordinate axes with origin O and the trajectory of a particle projected from O with speed $28\,\mathrm{m\,s^{-1}}$ at an angle α to the horizontal. After *t* seconds, the particle has horizontal and vertical displacements x m and y m.

Air resistance should be neglected.

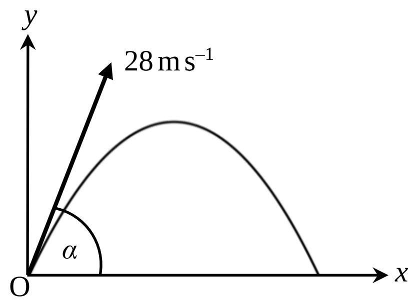

Fig. 12

(i) Show that the equation of the trajectory is given by

$$\tan^2\alpha - \frac{160}{x}\tan\alpha + \frac{160y}{x^2} + 1 = 0 . \qquad (*)$$ **[5]**

(ii) (*A*) Show that if (*) is treated as an equation with $\tan\alpha$ as a variable and with *x* and *y* as constants, then (*) has two distinct real roots for $\tan\alpha$ when $y < 40 - \frac{x^2}{160}$. **[3]**

(*B*) Show the inequality in part **(ii)** (*A*) as a locus on the graph of $y = 40 - \frac{x^2}{160}$ in the Printed Answer Booklet and label it R. **[1]**

S is the locus of points (x, y) where (*) has **one** real root for $\tan\alpha$.
T is the locus of points (x, y) where (*) has **no** real roots for $\tan\alpha$.

(iii) Indicate S and T on the graph in the Printed Answer Booklet. **[2]**

(iv) State the significance of R, S and T for the possible trajectories of the particle. **[3]**

A machine can fire a tennis ball from ground level with a maximum speed of $28\,\mathrm{m\,s^{-1}}$.

(v) State, with a reason, whether a tennis ball fired from the machine can achieve a range of 80 m. **[1]**

END OF QUESTION PAPER