

A Level Further Mathematics B (MEI) Y436 Further Pure with Technology Sample Question Paper

Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 45 minutes

You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:

- a scientific or graphical calculator
- Computer with appropriate software



INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

COMPUTING RESOURCES

- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages.

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Answer **all** the questions.

- 1** A family of curves has polar equation $r = \cos^n\left(\frac{\theta}{n}\right)$, $0 \leq \theta < n\pi$, where n is a positive *even* integer.
- (i) (A) Sketch the curve for the cases $n = 2$ and $n = 4$. [2]
- (B) State two points which lie on every curve in the family. [1]
- (C) State one other feature common to all the curves. [1]
- (ii) (A) Write down an integral for the length of the curve for the case $n = 4$. [2]
- (B) Evaluate the integral. [2]
- (iii) (A) Using $t = \theta$ as the parameter, find a parametric form of the equation of the family of curves. [1]
- (B) Show that $\frac{dy}{dx} = \frac{\sin t \sin\left(\frac{t}{n}\right) - \cos t \cos\left(\frac{t}{n}\right)}{\sin t \cos\left(\frac{t}{n}\right) + \cos t \sin\left(\frac{t}{n}\right)}$. [2]
- (iv) Hence show that there are $n + 1$ points where the tangent to the curve is parallel to the y -axis. [6]
- (v) By referring to appropriate sketches, show that the result in part (iv) is true in the case $n = 4$. [2]

- 2 (i) (A) Create a program to find all the solutions to $x^2 \equiv -1 \pmod{p}$ where $0 \leq x < p$.
Write out your program in full in the Printed Answer Booklet. [5]
- (B) Use the program to find the solutions to $x^2 \equiv -1 \pmod{p}$ for the primes
- $p = 809$,
 - $p = 811$ and
 - $p = 444001$. [3]
- (ii) State Wilson's Theorem. [1]
- (iii) The following argument shows that $(4k)! \equiv ((2k)!)^2 \pmod{p}$ for the case $p = 4k + 1$.
- $$(4k)! \equiv 1 \times 2 \times 3 \times \dots \times (2k-1) \times 2k \times (2k+1) \times (2k+2) \times \dots \times (4k-1) \times 4k \pmod{p} \quad (1)$$
- $$\equiv 1 \times 2 \times 3 \times \dots \times (2k-1) \times 2k \times (-2k) \times (-(2k-1)) \times \dots \times (-2) \times (-1) \pmod{p} \quad (2)$$
- $$\equiv ((2k)!)^2 \pmod{p} \quad (3)$$
- (A) Explain why $(2k+2)$ can be written as $(-(2k-1))$ in line (2). [1]
- (B) Explain how line (3) has been obtained. [2]
- (C) Explain why, if p is a prime of the form $p = 4k + 1$, then $x^2 \equiv -1 \pmod{p}$ will have at least one solution. [1]
- (D) Hence find a solution of $x^2 \equiv -1 \pmod{29}$. [2]
- (iv) (A) Create a program that will find all the positive integers n , where $n < 1000$, such that $(n-1)! \equiv -1 \pmod{n^2}$. Write out your program in full. [3]
- (B) State the values of n obtained. [2]
- (C) A Wilson prime is a prime p such that $(p-1)! \equiv -1 \pmod{p^2}$. Write down all the Wilson primes p where $p < 1000$. [1]

- 3 This question explores the family of differential equations $\frac{dy}{dx} = \sqrt{1+ax+2y}$ for various values of the parameter a . Fig. 3 shows the tangent field in the case $a = 1$.

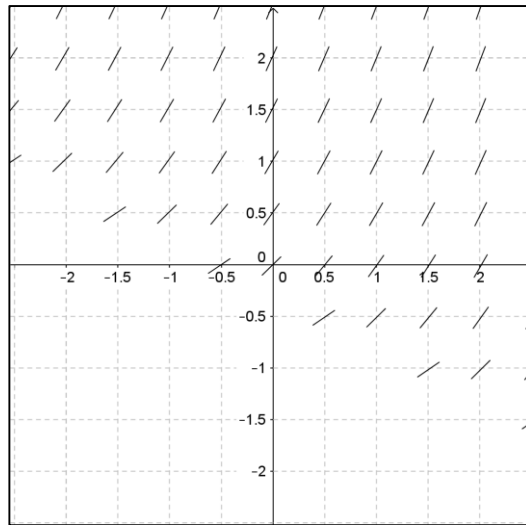


Fig. 3

- (i) (A) Sketch the tangent field in the case $a = -2$. [2]
- (B) Explain why the tangent field is not defined for the whole coordinate plane. [1]
- (C) Give an inequality which describes the region in which the tangent field is defined. [1]
- (D) Find a value of a such that the region for which the tangent field is defined includes the entire x -axis. [1]
- (ii) (A) For the case $a = 1$, with $y = 1$ when $x = 0$, construct a spreadsheet for the Runge-Kutta method of order 2 with formulae as follows, where $f(x, y) = \frac{dy}{dx}$.

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + h, y_n + k_1)$$

$$y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$$

State the formulae you have used in your spreadsheet. [3]

- (B) Use your spreadsheet to obtain the value of y correct to 4 decimal places when $x = 1$ for
- $h = 0.1$
- and
- $h = 0.05$. [2]

- (iii) (A) For the case $a=0$ find the analytical solution that passes through the point $(0, 1)$. [1]
- (B) Verify that the solution in part (iii) (A) is a solution to the differential equation. [2]
- (C) Use the solution in part (iii) (A) to find the value of y correct to 4 decimal places when $x=1$. [1]
- (iv) (A) Verify that $y = -\frac{a}{2}x + \frac{a^2}{8} - \frac{1}{2}$ is a solution for all cases when $a \leq 0$. [2]
- (B) Show that this is the only straight line solution in these cases. [4]

END OF QUESTION PAPER

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