OCR
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A Level Further Mathematics B (MEI)

Y420 Core Pure

Sample Question Paper Version 2

Date – Morning/Afternoon

Time allowed: 2 hours 40 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **144**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **24** pages. The Question Paper consists of **8** pages.

Section A (33 marks)

Answer **all** the questions.

1 Find the acute angle between the lines with vector equations $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$. **[3]**

2 **(i)** On an Argand diagram draw the locus of points which satisfy $\arg(z - 4\mathrm{i}) = \frac{\pi}{4}$. **[2]**

(ii) Give, in complex form, the equation of the circle which has centre at $6 + 4\mathrm{i}$ and touches the locus in part **(i)**. **[4]**

3 Transformation M is represented by matrix $\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$.

(i) On the diagram in the Printed Answer Booklet draw the image of the unit square under M. **[2]**

(ii) (*A*) Show that there is a constant k such that $\mathbf{M}\begin{pmatrix} x \\ kx \end{pmatrix} = 5\begin{pmatrix} x \\ kx \end{pmatrix}$ for all *x*. **[2]**

(*B*) Hence find the equation of an invariant line under M. **[1]**

(*C*) Draw the invariant line from part **(ii)** (*B*) on your diagram for part **(i)**. **[1]**

4 You are given that $z = 1 + 2\mathrm{i}$ is a root of the equation $z^3 - 5z^2 + qz - 15 = 0$, where $q \in \mathbb{R}$.

Find

- the other roots,
- the value of q. **[5]**

5 **(i)** Express $\frac{2}{(r+1)(r+3)}$ in partial fractions. **[2]**

(ii) Hence find $\sum_{r=1}^{n} \frac{1}{(r+1)(r+3)}$, expressing your answer as a single fraction. **[5]**

6 **(i)** A curve is in the first quadrant. It has parametric equations $x = \cosh t + \sinh t$, $y = \cosh t - \sinh t$ where $t \in \mathbb{R}$. Show that the cartesian equation of the curve is $xy = 1$. **[2]**

Fig. 6 shows the curve from part **(i)**. P is a point on the curve. O is the origin. Point A lies on the *x*-axis, point B lies on the *y*-axis and OAPB is a rectangle.

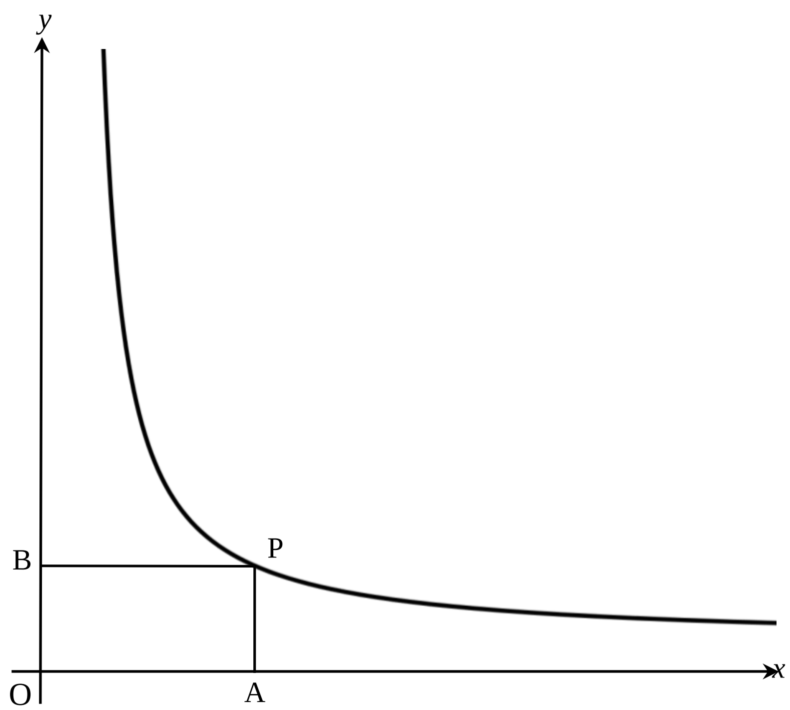

Fig. 6

(ii) Find the smallest possible value of the perimeter of rectangle OAPB. Justify your answer. **[4]**

Section B (111 marks)

Answer **all** the questions

7 **(i)** Use the Maclaurin series for $\ln(1+x)$ up to the term in x^3 to obtain an approximation to $\ln 1.5$. **[2]**

(ii) (*A*) Find the error in the approximation in part **(i)**. **[1]**

(*B*) Explain why the Maclaurin series in part **(i)**, with $x = 2$, should not be used to find an approximation to $\ln 3$. **[1]**

(iii) Find a cubic approximation to $\ln\left(\frac{1+x}{1-x}\right)$. **[2]**

(iv) (*A*) Use the approximation in part **(iii)** to find approximations to
- ln 1.5 and
- $\ln 3$. **[3]**

(*B*) Comment on your answers to part **(iv)** (*A*). **[2]**

8 Find the cartesian equation of the plane which contains the three points $(1, 0, -1)$, $(2, 2, 1)$ and $(1, 1, 2)$. **[5]**

9 A curve has polar equation $r = a\sin 3\theta$ for $-\frac{1}{3}\pi \leq \theta \leq \frac{1}{3}\pi$, where *a* is a positive constant.

(i) Sketch the curve. **[2]**

(ii) **In this question you must show detailed reasoning.**

Find, in terms of *a* and π, the area enclosed by one of the loops of the curve. **[5]**

10 **(i)** Obtain the solution to the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + 3y = \frac{1}{x}\text{, where } x > 0\text{,}$$

given that $y = 1$ when $x = 1$. **[7]**

(ii) Deduce that *y* decreases as *x* increases. **[2]**

11 **(i)** It is conjectured that

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots+\frac{n-1}{n!}=a-\frac{b}{n!},$$

where a and b are constants, and n is an integer such that $n \geq 2$.

By considering particular cases, show that if the conjecture is correct then $a=b=1$. **[2]**

(ii) Use induction to prove that

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots+\frac{n-1}{n!}=1-\frac{1}{n!} \text{ for } n \geq 2.$$ **[7]**

12 **In this question you must show detailed reasoning.**

(i) Given that $y=\arctan x$, show that $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+x^2}$. **[3]**

Fig. 12 shows the curve $y=\frac{1}{1+x^2}$.

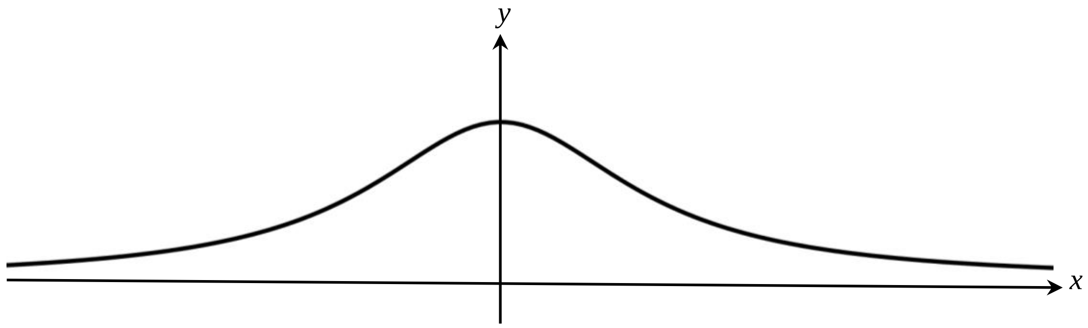

Fig. 12

(ii) Find, in exact form, the mean value of the function $\mathrm{f}(x)=\frac{1}{1+x^2}$ for $-1 \leq x \leq 1$. **[3]**

(iii) The region bounded by the curve, the x-axis, and the lines $x=1$ and $x=-1$ is rotated through 2π radians about the x-axis. Find, in exact form, the volume of the solid of revolution generated. **[7]**

13 Matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} k & 1 & -5 \\ 2 & 3 & -3 \\ -1 & 2 & 2 \end{pmatrix}$, where k is a constant.

(i) Show that $\det \mathbf{M} = 12(k-3)$. **[2]**

(ii) Find a solution of the following simultaneous equations for which $x \neq z$.

$$\begin{aligned} 4x^2 + y^2 - 5z^2 &= 6 \\ 2x^2 + 3y^2 - 3z^2 &= 6 \\ -x^2 + 2y^2 + 2z^2 &= -6 \end{aligned}$$

[3]

(iii) **(*A*)** Verify that the point $(2, 0, 1)$ lies on each of the following three planes.

$$\begin{aligned} 3x + y - 5z &= 1 \\ 2x + 3y - 3z &= 1 \\ -x + 2y + 2z &= 0 \end{aligned}$$

[1]

(*B*) Describe how the three planes in part **(iii)** (*A*) are arranged in 3-D space. Give reasons for your answer. **[4]**

(iv) Find the values of k for which the transformation represented by $\mathbf{M}$ has a volume scale factor of 6. **[3]**

14 **(i)** Starting with the result

$$e^{i\theta} = \cos\theta + i\sin\theta,$$

show that

(*A*) $\left(\cos\theta + i\sin\theta\right)^n = \cos n\theta + i\sin n\theta$ **[2]**

(*B*) $\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right).$ **[2]**

(ii) Using the result in part **(i)** (*A*), obtain the values of the constants *a*, *b*, *c* and *d* in the identity

$$\cos 6\theta \equiv a\cos^6\theta + b\cos^4\theta + c\cos^2\theta + d\,.$$ **[6]**

(iii) Using the result in part **(i)** (*B*), obtain the values of the constants *P*, *Q*, *R* and *S* in the identity

$$\cos^6\theta \equiv P\cos 6\theta + Q\cos 4\theta + R\cos 2\theta + S\,.$$ **[5]**

(iv) Show that $\cos\frac{\pi}{12} = \left(\frac{26 + 15\sqrt{3}}{64}\right)^{\frac{1}{6}}.$ **[3]**

15 **In this question you must show detailed reasoning.**

Show that

$$\int_0^{\frac{2}{3}} \operatorname{arsinh} 2x\,dx = \frac{2}{3}\ln 3 - \frac{1}{3}.$$ **[8]**

16 A small object is attached to a spring and performs oscillations in a vertical line. The displacement of the object at time t seconds is denoted by x cm.

Preliminary observations suggest that the object performs simple harmonic motion (SHM) with a period of 2 seconds about the point at which $x = 0$.

(i) (*A*) Write down a differential equation to model this motion. **[3]**

(*B*) Give the general solution of the differential equation in part **(i)** (*A*). **[1]**

Subsequent observations indicate that the object's motion would be better modelled by the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2k\frac{\mathrm{d}x}{\mathrm{d}t} + \left(k^2 + 9\right)x = 0 \qquad (*)$$

where k is a positive constant.

(ii) (*A*) Obtain the general solution of (*). **[3]**

(*B*) State two ways in which the motion given by this model differs from that in part **(i)**. **[2]**

The amplitude of the object's motion is observed to reduce with a scale factor of 0.98 from one oscillation to the next.

(iii) Find the value of k. **[3]**

At the start of the object's motion, $x = 0$ and the velocity is 12 $\mathrm{cm\,s^{-1}}$ in the positive x direction.

(iv) Find an equation for x as a function of t. **[4]**

(v) Without doing any further calculations, explain why, according to this model, the greatest distance of the object from its starting point in the subsequent motion will be slightly less than 4 cm. **[2]**

END OF QUESTION PAPER