Qualification Accredited

Oxford Cambridge and RSA

**A LEVEL**

*Examiners' report*

# *FURTHER MATHEMATICS B (MEI)*

**H645**

For first teaching in 2017

## Y431/01 Summer 2019 series

Version 1

# Contents

Introduction ..... 3
Paper Y431 series overview ..... 4
Question 1(a) ..... 5
Question 1(b) ..... 5
Question 1(c) ..... 5
Question 2(a) ..... 5
Question 2(b) ..... 6
Question 2(c) ..... 6
Question 2(d) ..... 6
Question 3 ..... 7
Question 4(a) ..... 8
Question 4(b) ..... 9
Question 4(c) ..... 9
Question 4(d) ..... 9
Question 5(a) ..... 10
Question 5(b) ..... 10
Question 5(c) ..... 11
Question 5(d) ..... 12
Question 5(e) ..... 12
Question 5(f) ..... 12
Question 6(a) ..... 13
Question 6(b) ..... 13
Question 6(c) ..... 14
Question 6(d) ..... 15

# Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects that caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

# Paper Y431 series overview

This was the first session for this mechanics Minor option, part of the new H645A Level specification Further Mathematics B (MEI).

Very few candidates provided a 'commentary' on their solutions. This would have been helpful for many, to focus their reasoning. Some candidates omitted to provide the physical explanations required following calculations.

A significant number of candidates did not appreciate that the 'show that' command requires a full and detailed explanation of all working.

| | | |
|---|---|---|
| i | **OCR support** | OCR's command words poster is available to download in both A2 and A4 versions from 'Assessment guides' at https://www.ocr.org.uk/qualifications/as-a-level-gce/further-mathematics-b-mei-h635-h645-from-2017/assessment. |

Most candidates were able to apply 'routine' methods successfully and overall demonstrated a comprehensive understanding of most of the material in the specification. Notably the application of couples and moments to solve problems was lacking in comparison, particularly the importance of considering moments in establishing equilibrium.

## Question 1(a)

**1** Dilip and Anna are doing an experiment to find the power at which they each work when running up a staircase at school. The top of the staircase is a vertical distance of 16 m above the bottom of the staircase.

Dilip, who has mass 75 kg, does the experiment first. Anna times him, and finds that he takes 5.6 seconds to run up the staircase.

**(a)** Find the average power generated by Dilip as he runs up the staircase. **[3]**

Nearly all candidates provided a fully complete solution with only a few making slips, such as neglecting to include $g$ in their calculation of work done.

## Question 1(b)

Anna, who has mass $M$ kg, then does the same experiment and runs up the staircase in 5.0 seconds. She works out that the average power she has generated is **less than** the corresponding value for Dilip.

**(b)** Find an inequality satisfied by $M$. **[2]**

Following completion of part (a), nearly all candidates produced an inequality that they then solved successfully.

## Question 1(c)

Gareth, who also has mass 75 kg, says that members of his sports club do an exercise similar to this, but they run up a 16 m high sand dune. Gareth can run up the sand dune in 8.4 seconds, but he claims that he generates more power than Dilip.

**(c)** Give a reason why Gareth's claim could be true. **[1]**

Most candidates thought this was due to additional friction or resistance rather than the additional vertical distance travelled due to the sand slipping away.

| | | |
|---|---|---|
| | **AfL** | Candidates should consider the question specifics to give an individual rather than 'generic' response when commenting on modelling assumptions. |

## Question 2(a)

**2** **(a)** Write down the dimensions of pressure. **[1]**

This was answered correctly by the majority of candidates. Occasionally units were given rather than the dimensions requested.

## Question 2(b)

The SI unit of pressure is the pascal (Pa). 15 Pa is equivalent to $Q$ newtons per square centimetre.

**(b)** Find the value of $Q$. **[1]**

The correct answer of 0.0015 was found by most, but 0.15 and 150000 were also often produced.

## Question 2(c)

Simon thinks the speed, $v$, of sound in a gas is given by the formula

$v = kP^x d^y V^z$,

where $P$ is the pressure of the gas,

$d$ is the density of the gas,

$V$ is the volume of the gas,

$k$ is a dimensionless constant.

**(c)** Use dimensional analysis to

- find the values of $x$ and $y$ and
- show that $z = 0$.

**[4]**

The routine approach was followed by very nearly all candidates successfully, except for the occasional slip in the dimensions of one the quantities. Very few who did not manage to show that $z$ = 0 rechecked and corrected their errors. A small number did not appreciate that 'show that' required a complete explanation.

| | | |
|---|---|---|
| | **AfL** | Make sure that 'show that' questions are answered with a full and complete explanation. |

## Question 2(d)

At normal atmospheric pressure the density of air at sea level is $1.29\,\text{kg m}^{-3}$. Under the same conditions the density of helium is $0.166\,\text{kg m}^{-3}$.

**(d)** Given that the speed of sound in air under these conditions is $340\,\text{m s}^{-1}$, use Simon's formula to find the speed of sound in helium under the same conditions. **[2]**

Most candidates completed this without problems. Some had difficulties using the reciprocal of the ratio of the square root of the two pressures, working out $340\sqrt{\frac{0.166}{1.29}}$ instead of $340\sqrt{\frac{1.29}{0.166}}$.

## Question 3

**3** Two identical uniform rectangular laminas, P and Q, each having length $ka$ and width $a$ are fixed together, in the same plane, to form a lamina R.
With reference to coordinate axes, the corners of P are at (0, 0), ($ka$, 0), ($ka$, $a$) and (0, $a$) and the corners of Q are at ($ka$, 0), ($ka + a$, 0), ($ka + a$, $ka$) and ($ka$, $ka$), as shown in Fig. 3.

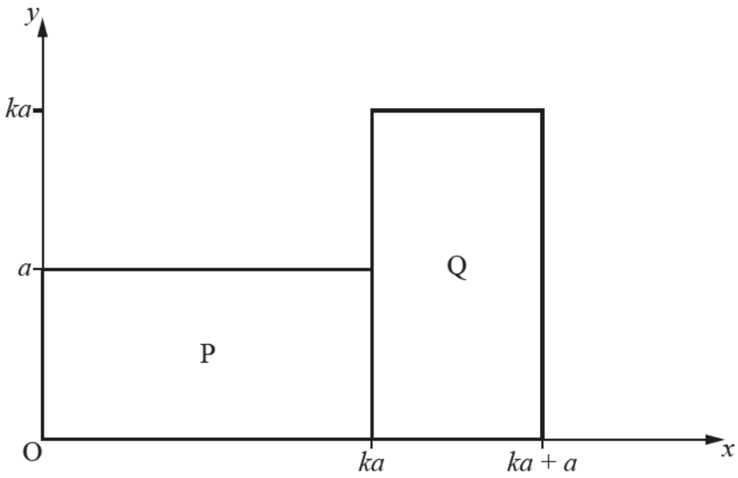


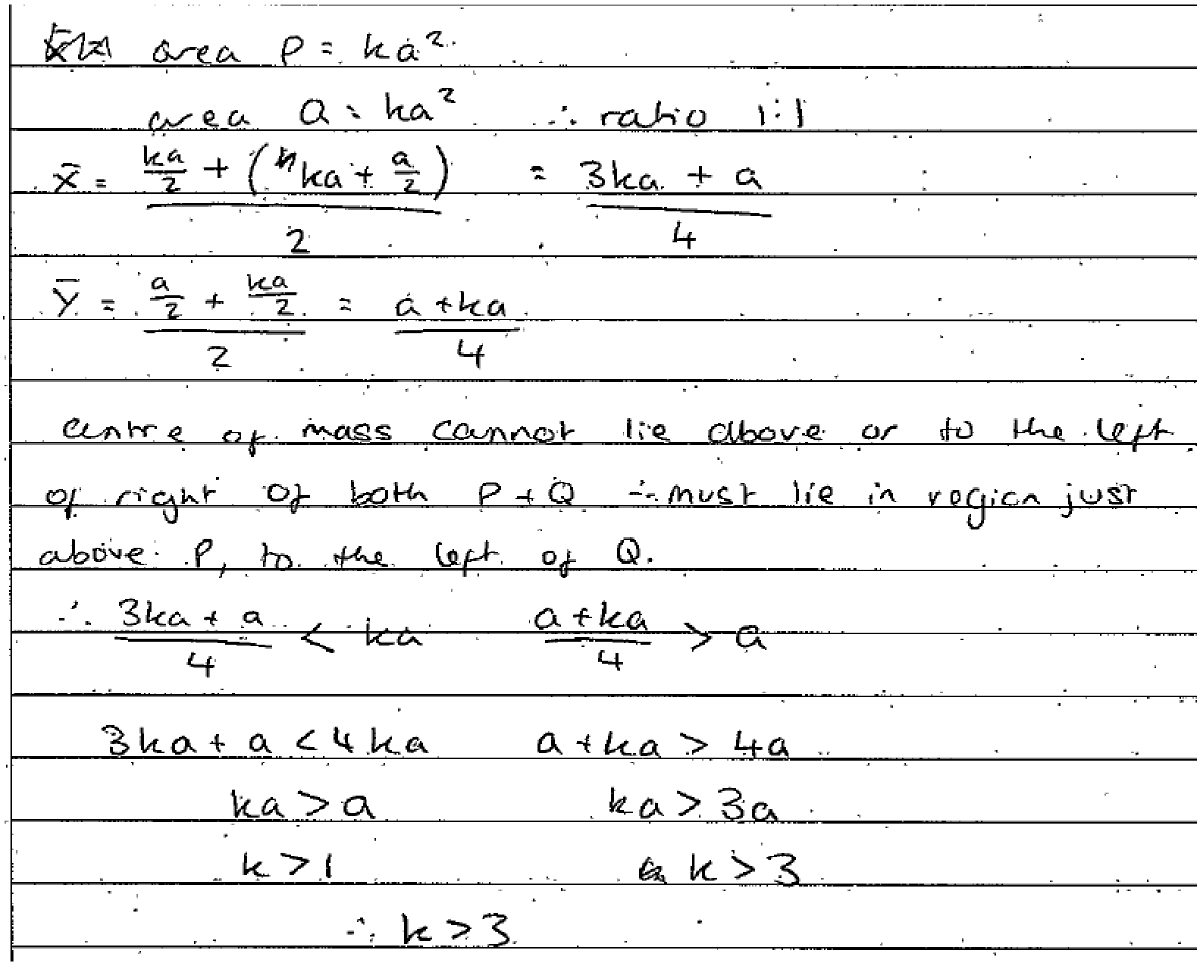

**Fig. 3**

Determine the range of values of $k$ for which the centre of mass of R lies outside the boundary of R. **[7]**

Nearly all candidates gained the first two marks for an attempt at finding the co-ordinates ($\bar{x}$, $\bar{y}$) of the centre of mass of the lamina R; a few obtained the wrong answers for these, with implications for later marks. The next two marks for the inequalities $\bar{x} < ka$ and $\bar{y} > a$ (defining the region outside the boundary of R) were gained by most although few candidates recognised this as the only possible region, with many adding other inequalities. The solution of the inequalities $k > 1$ and $k > 3$ invariably followed the correct inequalities and correct derivations of $\bar{x}$ and $\bar{y}$ . Occasionally the conclusion was not stated.

Very few had the clear explanation given in the exemplar shown below.

## Exemplar 1


~~x̄A~~ area P = $ka^2$
area Q = $ka^2$ ∴ ratio 1:1
$\bar{x} = \dfrac{\frac{ka}{2} + \left(ka + \frac{a}{2}\right)}{2} = \dfrac{3ka + a}{4}$
$\bar{y} = \dfrac{\frac{a}{2} + \frac{ka}{2}}{2} = \dfrac{a + ka}{4}$
centre of mass cannot lie above or to the left of right of both P + Q ∴ must lie in region just above P, to the left of Q.
∴ $\dfrac{3ka + a}{4} < ka$ $\dfrac{a + ka}{4} > a$
$3ka + a < 4ka$ $a + ka > 4a$
$ka > a$ $ka > 3a$
$k > 1$ $k > 3$
∴ $k > 3$

## Question 4(a)

**4** Two model railway trucks, A of mass 0.1 kg and B of mass 0.2 kg, are constrained to move on a smooth straight level track.
Initially B is stationary and A is moving towards B with speed $u\,\mathrm{m\,s^{-1}}$ before they collide. The coefficient of restitution between A and B is $e$.

**(a)** Find the speed of A and the speed of B after the collision, giving your answers in terms of $e$ and $u$. **[5]**

This was a routine collision question that was completed successfully by most candidates. Errors included stating $e = \dfrac{\text{speed of approach}}{\text{speed of separation}}$ and $e = \dfrac{\text{final momentum}}{\text{initial momentum}}$.

## Question 4(b)

**(b)** Show that the loss of kinetic energy in the collision is $\frac{1}{30}u^2(1-e^2)$. **[2]**

Many candidates did not successfully show the given result. This was due to lack of sufficiently detailed explanation, notably missing the expansion $V_A^2$ and $V_B^2$.

| | | |
|---|---|---|
| | **AfL** | Candidates should make sure that 'show that' questions are answered with a full and complete explanation. |

## Question 4(c)

**(c)** For the case in which the **loss** of kinetic energy is **least**

- state the value of $e$
- state the loss in kinetic energy
- describe the subsequent motion of the trucks. **[3]**

Most candidates stated $e = 1$ and $\Delta E = 0$, but many did not use the expressions from part (a) to calculate final velocities.

## Question 4(d)

**(d)** For the case in which the **loss** of kinetic energy is **greatest**

- state the value of $e$
- state the loss in kinetic energy
- describe the subsequent motion of the trucks. **[3]**

Most candidates stated $e = 1$ and $\Delta E = \frac{1}{30}u^2$, but many did not use the expressions from part (a) to calculate the final velocity and fewer recognised that the trucks coalesced.

## Question 5(a)

**5** Jack and Jemima are pulling a boat along a straight level canal.
The resistance to the motion of the boat is modelled as constant and equal to 1200 N.
Jack and Jemima walk in the same direction on paths on opposite sides of the canal. They each walk forwards at the same steady speed, keeping level with each other so that the distance between them is always 6 m. Jack and Jemima each pull a long light inextensible rope attached to the boat; initially they hold their ropes so the distance from each of them to the boat is 5 m, as shown in Fig. 5.1.

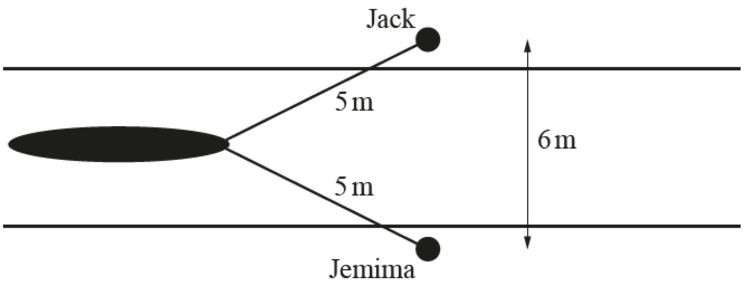



**Fig. 5.1**

**(a)** Explain why the tension will be the same in each rope. **[1]**

Very few candidates stated this was due to symmetry and instead produced incomplete equivalent explanations. A very small number explained that the vertical components acting on the boat must be equal and opposite and that the angles made by the ropes with the bank were equal.

## Question 5(b)

**(b)** Find the tension in each rope. **[3]**

Almost all candidates found the tension in each rope to be 750 N.

## Question 5(c)

Jemima then gradually releases more rope, so that the distance between her and the boat is 7 m. Jack and Jemima continue to walk at the same steady speed along the paths, but the position of the boat changes so that Jemima's rope makes an angle of $\theta$ with the path and Jack's rope makes an angle of $\phi$ with the path, as shown in Fig. 5.2.

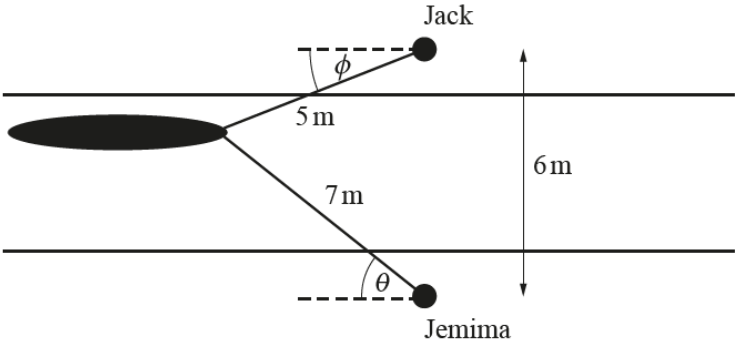
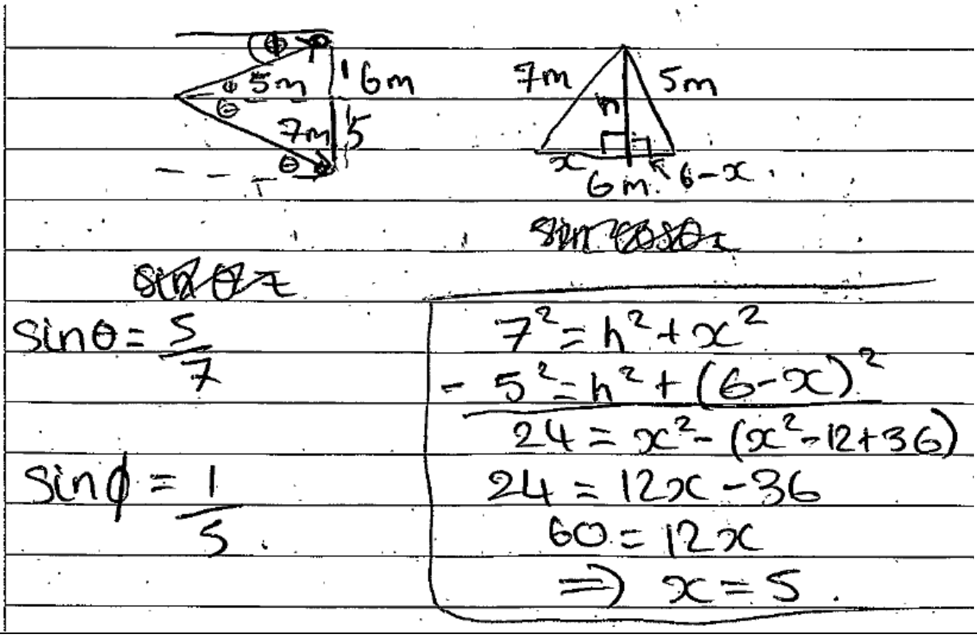



**Fig. 5.2**

**(c)**
- Show that $\sin\phi = \frac{1}{5}$.
- Show that $\sin\theta = \frac{5}{7}$. **[2]**

A significant number of candidates did not appreciate the use of 'show that' and either simply worked the question backwards or produced a circular argument. This was completed successfully mainly by application of the cosine rule (as shown in the mark scheme) or solving the simultaneous equations

$x^2 + a^2 = 25$ and $(6 - x)^2 + a^2 = 49$ to yield $x = 1$ and thereby finding $\sin \varnothing = \frac{1}{5}$ and $\sin\theta = \frac{6-1}{7} = \frac{5}{7}$.

| | | |
|---|---|---|
| | **AfL** | Make sure that 'show that' questions are answered with a full and complete explanation (as in the exemplar answer below).<br><br>Exemplar 2<br><br> |



## Question 5(d)

**(d)** Find the tension in each rope in this new equilibrium position. **[5]**

This was completed successfully by solving the set of equilibrium equations $T_1 \cos \varnothing + T_2 \cos \theta = 1200$ and $T_1 \sin \varnothing = T_2 \sin \theta$. Solutions were occasionally marred by algebraic errors, notably a number could not manipulate the equation $\frac{T_1}{5} = \frac{5T_2}{7}$ correctly to find $T_1$ in terms of $T_2$. Use of Lami's theorem was seen a couple of times and a number used calculators to solve the simultaneous equations directly.

## Question 5(e)

**(e)** Without further calculation, state the effect on the tensions in the ropes if Jack now lengthens his rope to 7 m, the same length as Jemima's rope. **[2]**

The most common answer was that Jack's tension decreased while Jemima's tension increased. Very few stated that the tensions would become equal and fewer that they would be less than 750N.

| | | |
|---|---|---|
| | **AfL** | Make sure explanations of situations are complete, describing all detail, using the number of marks allocated as a guide. |

## Question 5(f)

**(f)** Suggest how the modelling assumption used in this question could be improved. **[1]**

Most correctly commented that the resistance should be variable

## Question 6(a)

**6** A uniform solid cylinder, L, has base radius 5 cm, height 24 cm and mass 5 kg.
L is placed on a rough plane inclined at an angle $\alpha$ to the horizontal, as shown in Fig. 6.

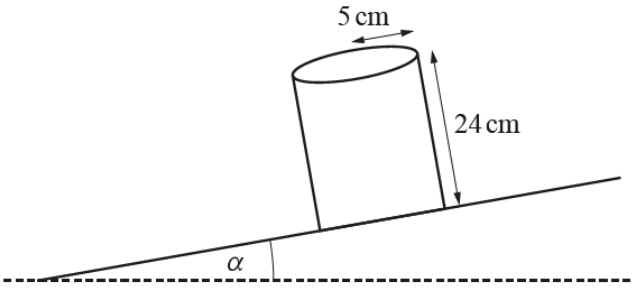



**Fig. 6**

**(a)** On the copy of Fig. 6 in the Printed Answer Booklet mark the forces acting on L. **[1]**

The diagram was often well done, although some omitted a required force. The direction of some forces could have been clearer.

| | | |
|---|---|---|
| | **AfL** | Make sure force diagrams are very clear, particularly the direction of the forces. If additional forces such as weight components are drawn, use dashed lines to make this clear. |

## Question 6(b)

The coefficient of friction between L and the plane is 0.3. Initially $\alpha$ is 15°.

**(b)** Show that L rests in equilibrium on the plane. **[4]**

Most candidates considered the sliding situation only, from which they deduced that the cylinder had to be in equilibrium. Very few considered the toppling situation.

| | | |
|---|---|---|
| | **AfL** | For statements of equilibrium on a plane, consideration of sliding **and** toppling are required. |

## Question 6(c)

A couple is applied to L. It is given that L will topple if the couple is applied in an anticlockwise sense, but L will not topple if the couple is applied in a clockwise sense.

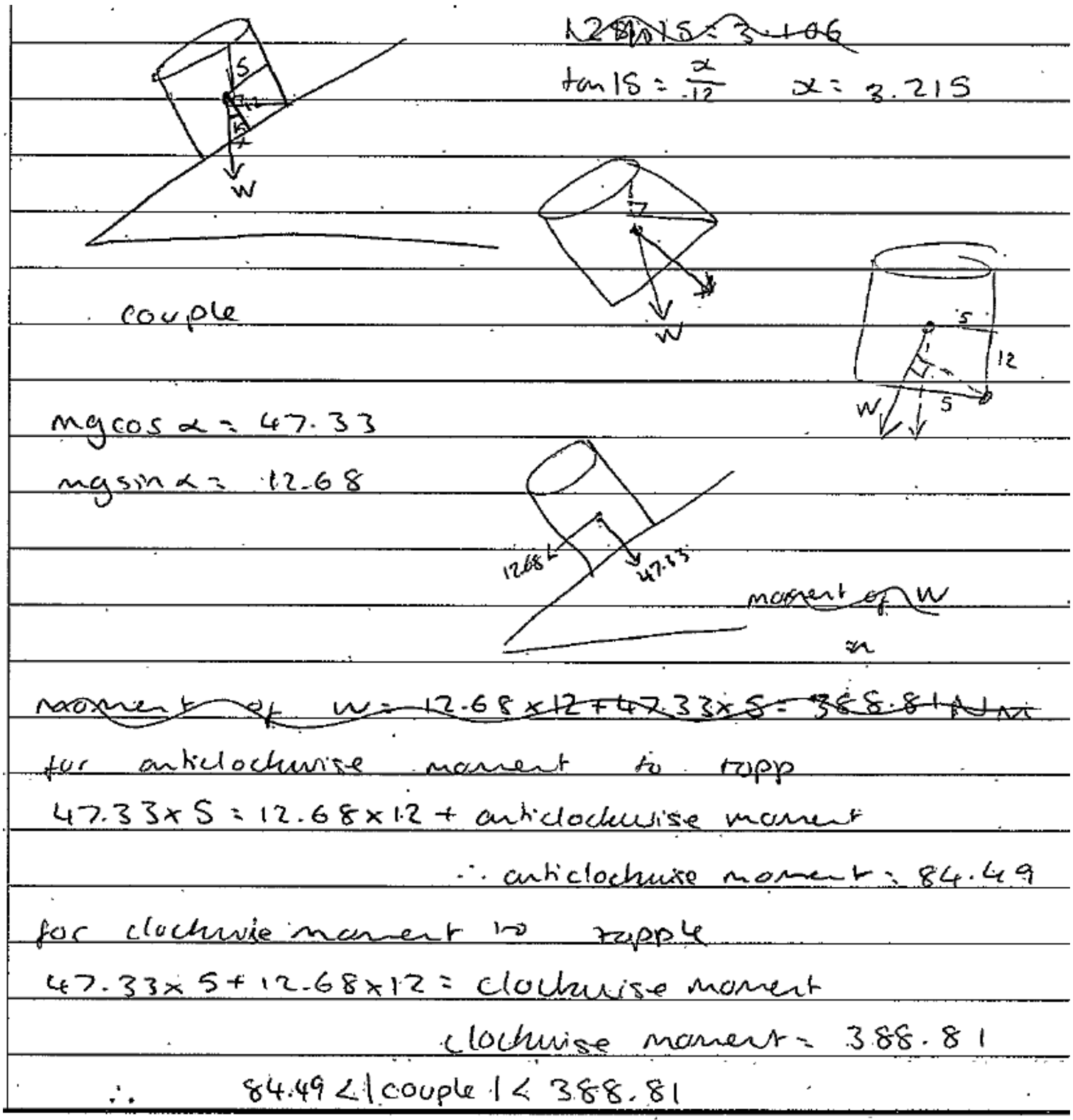
**(c)** Find the range of possible values of the magnitude of the couple. **[4]**

Very few managed to write correct moment equations using both components of the weight $5g(12\sin 15°)$ and $5g(5\cos 15°)$.

**AfL**

A clear diagram and commentary are helpful to the candidate in constructing a clear solution as shown in Exemplar 3 below.

### Exemplar 3



## Question 6(d)

The couple is now removed and the plane is slowly tilted so that $\alpha$ increases.

**(d)** Determine whether L topples first without sliding or slides first without toppling. **[3]**

This was successfully done by many candidates, finding that the sliding angle (16.7°) was less than the tipping angle (22.6°). Errors usually occurred when calculating the condition for toppling, often incorrect values for the ratio of dimensions equal to tan *α*. Occasionally concluding statements were not made.

| | | |
|---|---|---|
| | **AfL** | For statements of equilibrium, comparison of the values calculated with a concluding statement is required. |

# Supporting you

For further details of this qualification please visit the subject webpage.

## Review of results

If any of your students' results are not as expected, you may wish to consider one of our review of results services. For full information about the options available visit the OCR website. If university places are at stake you may wish to consider priority service 2 reviews of marking which have an earlier deadline to ensure your reviews are processed in time for university applications.

activeresults

***Review students' exam performance with our free online results analysis tool. Available for GCSE, A Level and Cambridge Nationals.***

It allows you to:

- review and run analysis reports on exam performance
- analyse results at question and/or topic level*
- compare your centre with OCR national averages
- identify trends across the centre
- facilitate effective planning and delivery of courses
- identify areas of the curriculum where students excel or struggle
- help pinpoint strengths and weaknesses of students and teaching departments.

*To find out which reports are available for a specific subject, please visit ocr.org.uk/administration/support-and-tools/active-results/

Find out more at ocr.org.uk/activeresults

# CPD Training

Attend one of our popular CPD courses to hear exam feedback directly from a senior assessor or drop in to an online Q&A session.

Please find details for all our courses on the relevant subject page on our website.

www.ocr.org.uk

The small print

**OCR Resources:** *the small print*

OCR's resources are provided to support the delivery of OCR qualifications, but in no way constitute an endorsed teaching method that is required by OCR. Whilst every effort is made to ensure the accuracy of the content, OCR cannot be held responsible for any errors or omissions within these resources. We update our resources on a regular basis, so please check the OCR website to ensure you have the most up to date version.

This resource may be freely copied and distributed, as long as the OCR logo and this small print remain intact and OCR is acknowledged as the originator of this work.

Our documents are updated over time. Whilst every effort is made to check all documents, there may be contradictions between published support and the specification, therefore please use the information on the latest specification at all times. Where changes are made to specifications these will be indicated within the document, there will be a new version number indicated, and a summary of the changes. If you do notice a discrepancy between the specification and a resource please contact us at: resources.feedback@ocr.org.uk.

Whether you already offer OCR qualifications, are new to OCR, or are considering switching from your current provider/awarding organisation, you can request more information by completing the Expression of Interest form which can be found here: www.ocr.org.uk/expression-of-interest

Please get in touch if you want to discuss the accessibility of resources we offer to support delivery of our qualifications: resources.feedback@ocr.org.uk

## Looking for a resource?

There is now a quick and easy search tool to help find **free** resources for your qualification:

www.ocr.org.uk/i-want-to/find-resources/

**www.ocr.org.uk**

OCR Customer Support Centre

**General qualifications**

Telephone 01223 553998
Facsimile 01223 552627
Email general.qualifications@ocr.org.uk



Cambridge Assessment

CERTIFIED ISO 9001 UKAS MANAGEMENT SYSTEMS 001