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AS LEVEL

Examiners' report

FURTHER MATHEMATICS A

H235
For first teaching in 2017

Y531/01 Summer 2019 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the exam paper can be downloaded from OCR.

Paper Y531 series overview

Y531 is the mandatory paper for AS Further Mathematics. It is taken alongside two other papers which can be freely chosen from a choice of 4. It tests knowledge of proof, complex numbers, matrices, vectors and further algebra as well as testing the understanding of the overarching themes of mathematical argument, problem solving and modelling. To do well on this paper candidates need a thorough understanding of the techniques covered and they need to support their answers with detailed working. They also need to have good algebraic and numerical manipulation skills.

Candidates generally seemed to be well prepared for this paper, and all but a very small number had sufficient time to attempt all the questions. Presentation was generally good with working well laid out, although there were a few candidates whose could have made their solutions clearer.

Candidates did well on the routine application of methods, such as in Questions 1(a), 2(b) and 3 parts (b) and (c). Question 8 (proof by induction) was also done well.

Candidates found questions which asked them to apply their knowledge in slightly more unfamiliar situations more difficult, such as in Questions 3(e) and 7(b) which had high rates of omission. In particular the concept of 'coplanar' was found difficult by many candidates.

Candidates must take careful note of any 'command words' given in the question. In particular, if a question asks for 'detailed reasoning', candidates must show all of their working. Calculators can be used as a check, but the solutions must be fully detailed.

i	**OCR support**	A poster detailing the different command words and what they mean is available here: https://teach.ocr.org.uk/italladdsup

There were very few cases of candidates writing solutions in the wrong answer box, and those who did usually indicated which question there were actually answering. It is very important that candidates who mistakenly write their solutions in the wrong space clearly identity this to the examiner.

Multiple solutions to questions were rare, with candidates usually deleting any work they did not want marked leaving one solution. However, candidates should note that in the situation when 2 or more solutions are made then it is the last solution that will be marked, even if this results in the candidate gaining little credit.

The 'Additional Answer Space' at the end of the answer booklet (page 12) was used frequently. Mostly this was for a continuation of Question 8 (the last question), but sometimes for second attempts at other questions. Candidates should take note of the instruction at the top of this page which says 'The question number(s) must be clearly shown in the margin(s)' so that examiners can identify which question the working should be attached to.

Question 1 (a)

1 You are given that $z = 3 - 4\mathrm{i}$.

(a) Find

- $|z|$,
- $\arg(z)$,
- z^*. **[3]**

The majority of candidates gained full credit for this question part. The ones who did not gain full marks usually did not gain the mark for finding arg(*z*). This tended to be through failure to include the negative sign, and sometimes through using arctan(3/4) rather than arctan(4/3).

Question 1 (b)

On an Argand diagram the complex number w is represented by the point A and w^* is represented by the point B.

(b) Describe the geometrical relationship between the points A and B. **[2]**

This was another question part where candidates did well. The majority correctly identified that the relationship was a reflection in the *x* axis. A few erroneously identified the reflection line as being the *y* axis.

Some candidates chose to describe the geometrical relationship using terms other than reflection, usually by considering the triangle made by the points *A*, *B* and the origin. For these candidates to gain full credit they had to describe the relationship in enough detail to imply that there is a reflection in the *x* axis.

No credit was given to candidates who did not describe the relationship in geometrical terms, usually these candidates stated that the real parts were the same but the imaginary parts are the negative of each other.

Question 2 (a)

2 Matrices **P** and **Q** are given by $\mathbf{P} = \begin{pmatrix} 1 & k & 0 \\ -2 & 1 & 3 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} (1+k) & -1 \end{pmatrix}$ where k is a constant.

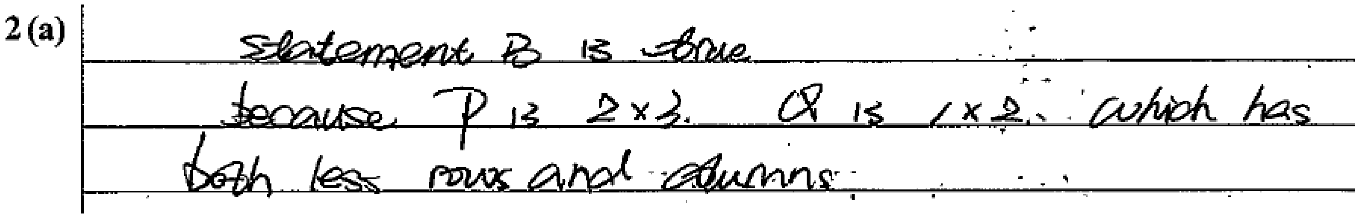
Exactly one of statements A and B is true.

Statement A: **P** and **Q** (in that order) are conformable for multiplication.
Statement B: **Q** and **P** (in that order) are conformable for multiplication.

(a) State, with a reason, which **one** of A and B is true. **[2]**

Almost all candidates could identify that statement B was the true statement. Most then also went on to give a correct reason for this usually by stating that the number of rows of P is equal to the number of columns of Q. Some candidates used the dimensions of the matrices to justify which statement was true, this method was usually used well but occasionally it was not clear which two numbers must be the same.

Exemplar 1


2 (a) Statement B is true because P is 2 x 3. Q is 1 x 2. which has both less rows and columns.

This candidate has correctly identified the dimensions of the two matrices but has not shown which two numbers must be the same for the matrices to be conformable for multiplication. Only the first mark given as the reason was not clear enough.

Question 2 (b)

(b) Find either **PQ** or **QP** in terms of k. **[2]**

This question was answered well with the majority of candidates gaining both marks. The most common mistakes were to miscopy the sign of one of the elements of the two matrices, or to express the matrix QP as a 3 x 1 matrix rather than a 1 x 3 matrix. A small number of candidates attempted to find PQ.

Question 3 (a)

3 The position vector of point A is $\mathbf{a} = -9\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$.
The line l passes through A and is perpendicular to $\mathbf{a}$.

(a) Determine the shortest distance between the origin, O, and l. **[2]**

The majority of candidates gained full credit for this question. Those who did not tended to not realise that the needed to find the length *OA* and tried to find vectors perpendicular to **a**, or wrote down **a** and made no further progress.

Question 3 (b)

l is also perpendicular to the vector $\mathbf{b}$ where $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$.

(b) Find a vector which is perpendicular to both **a** and **b**. **[1]**

This question was answered well by the majority of candidates. A large number used their calculator efficiently to find the required vector. Candidates who found an incorrect vector were usually showing their working on paper and made a sign mistake when trying to find the cross product.

Question 3 (c)

(c) Write down an equation of l in vector form. **[1]**

	AfL	Candidates are often unclear on the difference between an equation, an identity and an expression. In this question one of the most common reasons for candidates to not gain the mark was that they omitted '**r** = …', and so did not give their answer as an equation. This was also a common error in Question 5b (although less common than in this question).

Fewer candidates gained this mark as compared to part (b), despite follow through being available for those who incorrectly answered part (b). Apart from writing an expression rather than an equation, the most common mistake was to use **b** – **a** as the direction of the line *l*.

Question 3 (d)

P is a point on l such that $PA = 2OA$.

(d) Find angle POA giving your answer to 3 significant figures. **[3]**

This question part was found to be very difficult, with over half of candidates gaining no credit. 9% of candidates made no attempt at this question part. The most common mistake was to use $\overrightarrow{PA} = 2\overrightarrow{OA}$ i.e. that the vector $\overrightarrow{PA}$ is equal to $2\overrightarrow{OA}$ rather than the length *PA* is twice the length of *OA*. The minority of candidates who did correct identify that PAO is a right angled triangle generally went on to gain full credit.

A few candidates tried to find the vector $\overrightarrow{OP}$ by using a general point on *l* and using the fact that the length of *PA* was twice the length of *OA*. Those who went on to find that this gave $\lambda = \pm\frac{11\sqrt{2}}{5}$ usually went on to gain full credit.

Question 3 (e)

C is a point whose position vector, $\mathbf{c}$, is given by $\mathbf{c} = p\mathbf{a}$ for some constant p. The line m passes through C and has equation $\mathbf{r} = \mathbf{c} + \mu\mathbf{b}$. The point with position vector $9\mathbf{i} + 8\mathbf{j} - 12\mathbf{k}$ lies on m.

(e) Find the value of p. **[3]**

This question part was also found to be difficult, and 23% of candidates did not attempt it. However, more candidates gained full credit as compared to the previous part. Those candidates who set up the correct vector equation almost always went on to solve the equation correctly.

The high omission rate might be partially due to the fact that 3(d) was found to be difficult. Candidates should always look at all the question parts as later ones might still be accessible even if earlier ones were found to be difficult.

Question 4 (a)

4 In this question you must show detailed reasoning.

You are given that $f(z) = 4z^4 - 12z^3 + 41z^2 - 128z + 185$ and that $2+i$ is a root of the equation $f(z) = 0$.

(a) Express $f(z)$ as the product of two quadratic factors with integer coefficients. **[5]**

	AfL	In this question detailed working is required, so candidates should make sure that they justify their working fully in all of the parts of the question. Using a calculator to solve quadratics etc. is only advisable as a check after the solutions have been found algebraically

Part (a) was answered well with over 60% of candidates gaining full marks here, and the most common reason for candidates to not gain full credit was to not showing working when expanding $\left(z-(2+i)\right)\left(z-(2-i)\right)$. Various method were used to find the second quadratic factor and algebraic division, coefficient matching and inspection were all used well. Candidates who found the other two roots using their calculator and expanded these related factors usually did not find a second quadratic factor with integer coefficients.

Question 4 (b)

(b) Solve $f(z) = 0$. **[3]**

Many candidates here did not show working to justify their roots of $\frac{-1 \pm 6i}{2}$, which is probably because they used their calculators to solve the quadratic or original quartic. Candidates must make sure that if 'detailed reasoning' is required in the stem of the question that they show working throughout the question. A special case was made to allow candidates who wrote down all 4 correct roots one mark.

Question 4 (c)

Two loci on an Argand diagram are defined by $C_1 = \{z : |z| = r_1\}$ and $C_2 = \{z : |z| = r_2\}$ where $r_1 > r_2$. You are given that two of the points representing the roots of $f(z) = 0$ are on C_1 and two are on C_2. R is the region on the Argand diagram between C_1 and C_2.

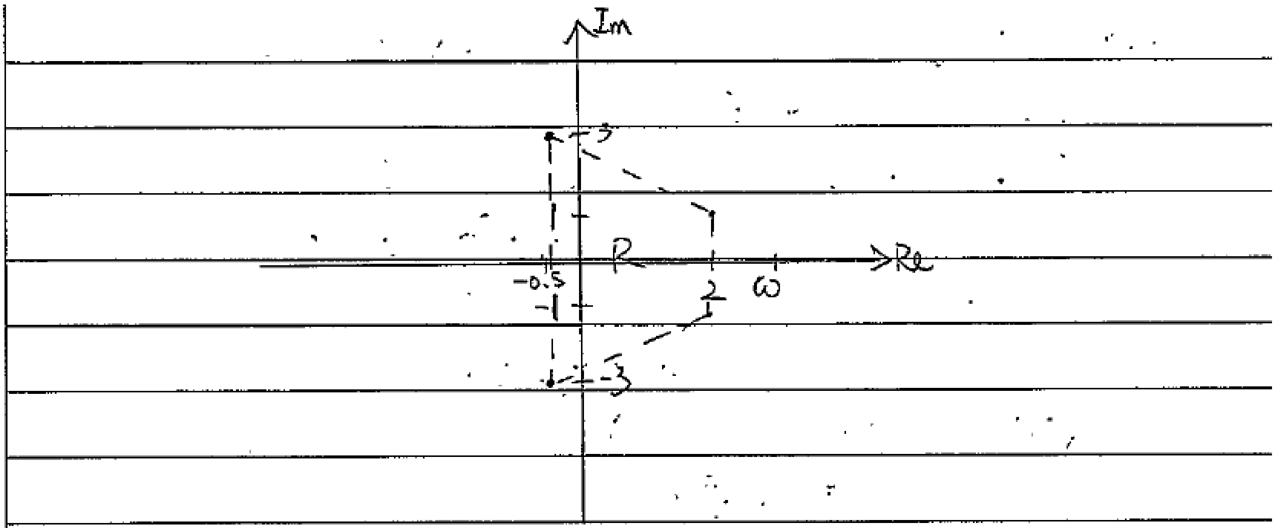
(c) Find the exact area of R. **[4]**

This question was found to be difficult with 16% of candidates making no attempt at it. The most common mistake was to plot the 4 roots of the quartic and then find the area of the trapezium formed by these.

The notation of the loci seemed to be unfamiliar to some candidates. Those candidates who recognised the loci as two concentric circles usually went on to gain full credit.

Exemplar 2

4(c)



upper base : 1 + 1 = 2
lower base : 3 + 3 = 6
height : 2 + 0.5 = 2.5
$\therefore R = \frac{(2+6)\cdot 2.5}{2}$
$= 10$

This is a typical example of the most common mistake made for this question part.

Question 4 (d)

(d) ω is the sum of all the roots of $f(z) = 0$.

Determine whether or not the point on the Argand diagram which represents ω lies in R. **[2]**

This question was omitted by 19% of candidates, which is probably mainly due to part (c) being found difficult. The key word 'Determine', along with the request for detailed reasoning at the start of the question meant that candidates were expected to fully justify their answer. The most common ways for candidates to fail to gain credit were not showing how they obtained $\omega = 3$ or by not stating that 3 lies between $\sqrt{5}$ and $\frac{\sqrt{37}}{2}$ - often only one of the two radii was compared to 3.

Question 5 (a)

5 In this question you must show detailed reasoning.

You are given that α, β and γ are the roots of the equation $5x^3 - 2x^2 + 3x + 1 = 0$.

(a) Find the value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. **[5]**

There were a lot of very good answers to this question and very few candidates failed to gain any credit at all. Using the formulae for the sums and products of roots was the most popular method, but many candidates used a substitution of $x = \sqrt{u}$ efficiently and effectively. Some candidates were unsure on how to deal with their root terms and tried squaring each term individually.

The most common mistakes were making a sign error when finding the sum and product ratios, and from either expanding $(\alpha\beta + \beta\gamma + \gamma\alpha)^2$ incorrectly or from not factorising $2\alpha^2\beta\gamma + 2\alpha\beta^2\gamma + 2\alpha\beta\gamma^2$. Omitting the factor of 2 was also a reasonably common mistake.

Question 5 (b)

(b) Find a cubic equation whose roots are α^2, β^2 and γ^2 giving your answer in the form $ax^3 + bx^2 + cx + d = 0$ where a, b, c and d are integers. **[4]**

There were more candidates who used a substitution method for this part as compared to previous part, and substitution was the more common method. Most candidates who used this could apply it well, but some were unsure how to deal with the root terms. Those that correctly gathered and squared sometimes made the mistake $\left(5u\sqrt{u}\right)^2 = 5u^3$. Some candidates found the cubic correctly, but then did not write this as an equation i.e. they omitted the '=0', and so did not gain the last mark.

Some candidates who had already found the equation in part (a) – usually through using the substitution method – did not write much working in this part. Detailed reasoning was requested, but candidates were allowed to refer back to their previous working and some credit was given for re-writing their correct cubic. An appendix of different situations for this question is given at the back of the mark scheme.

Question 6

6 A transformation T is represented by the matrix **T** where $\mathbf{T} = \begin{pmatrix} x^2+1 & -4 \\ 3-2x^2 & x^2+5 \end{pmatrix}$.

A quadrilateral Q, whose area is 12 units, is transformed by T to Q'.

Find the smallest possible value of the area of Q'. **[5]**

The most common mark given for this question was 2 (usually for candidates who found the determinant correctly), followed by 5. Some candidates found the determinant correctly but then were not sure how to find the smallest value of this. The most common mistakes were sign errors when finding the determinant, assuming that the determinant would be minimised when $x = 0$ or trying to solve det(T) = 0.

The most common (correct) method to try to find the minimum of the determinant was to differentiate, although this did lead some candidates to choose the incorrect $x = 0$. Completing the square was used fluently in some cases.

Question 7 (a)

7 A transformation A is represented by the matrix **A** where $\mathbf{A} = \begin{pmatrix} -1 & x & 2 \\ 7-x & -6 & 1 \\ 5 & -5x & 2x \end{pmatrix}$.

The tetrahedron H has vertices at O, P, Q and R. The volume of H is 6 units.

P', Q', R' and H' are the images of P, Q, R and H under A.

(a) In the case where $x = 5$

- find the volume of H',
- determine whether A preserves the orientation of H. **[3]**

A fairly common mistake was to multiply 5 by det(A) rather than the original volume of *H* (which was 6). Other mistakes included $\text{Vol}(H') = 6 \times 80^3$ or $\text{Vol}(H') = 80^6$. Some candidates gave a negative volume as their answer. Follow through on the sign of their determinant was allowed for the final E mark regarding the orientation of the tetrahedron.

Some candidates found det(A) before substituting *x* = 5, which did lead to some making algebraic mistakes before their substitution. In this part, candidates who used their calculator to find det(A) were often more successful than candidates who worked it out longhand.

Question 7 (b)

(b) Find the values of x for which O, P', Q' and R' are coplanar (i.e. the four points lie in the same plane). **[4]**

?	**Misconception**	Many candidates were unsure of the term 'coplanar', and some thought it means that the points has to line on one of the planes *x* = 0, *y* = 0 or *z* = 0.

This question part has the highest rate of omission with 38% of candidates not attempting it, and it was found to be the hardest question part on the paper. Many candidates did not seem to understand the connection between the points being coplanar and the volume of the tetrahedron being zero.

Quite a few candidates who attempted to find the determinant made sign errors, or algebraic and arithmetical errors (including frequently $1 \times 5 = 6$).

Candidates who found the correct cubic equation almost always went on to find the correct values of *x*, many using their calculators accurately to do this.

Question 8

8 **In this question you must show detailed reasoning.**

M is the matrix $\begin{pmatrix} 1 & 6 \\ 0 & 2 \end{pmatrix}$.

Prove that $\mathbf{M}^n = \begin{pmatrix} 1 & 3(2^{n+1}-2) \\ 0 & 2^n \end{pmatrix}$, for any positive integer n. **[6]**

This question was generally answered well, and candidates appeared to be confident with the idea of proof by induction.

The most common mistakes were:

* Not finishing off the proof by induction correctly, usually by stating 'since it is true for n = 1, n = k and n = k+1 …'

* Failing to show sufficient detail to justify the n = 1 case

* Failing to show sufficient detail to show that the n = k+1 case is true given that the n = k case is true

* Confusion when multiplying 2×2^k which often was put equal to 4^k

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