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Examiners' report

MATHEMATICS A

H240
For first teaching in 2017

H240/03 Summer 2018 series

Version 1

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Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper H240/03 series overview

This is the third examination component for the new revised A-Level examination for GCE Mathematics A. It is a two hour paper consisting of 100 marks which tests content from Pure Mathematics (Section A, 50 marks) and Mechanics (Section B, 50 marks). Pure Mathematics content is tested on all three papers, and any topic could be tested on any of the three papers.

Inevitably, the report that follows will concentrate on aspects of the candidates' performance where improvement is possible to assist centres on preparing candidates for future series. However, this should not obscure the fact that a significant number of candidates who sat this first paper in this reformed A Level qualification produced solutions which were a pleasure for examiners to assess. Many candidates demonstrated a most impressive level of mathematical ability and insight which enabled them to meet the various challenges posed by this paper on both the pure and mechanics content; precision, command of correct mathematical notation and excellent presentational skills were evident in many scripts. The majority of candidates in this first series have studied A Level Maths in one year and plan to study Further Maths next year.

The specification includes some guidance about the level of written evidence required in assessment question; these were provided to reflect the increased functionality of the available calculators and the changes in assessment objectives, since there is a significant change from when the equivalent legacy qualifications were designed.

There are a number of questions on this paper began with the demand '**In this question you must show detailed reasoning**'; to quote the specification that 'when a question includes this instruction candidates must give a solution which leads to a conclusion showing a detailed and complete analytical method. Their solution should contain sufficient detail to allow the line of their argument to be followed. This is not a restriction on a candidate's use of a calculator when tackling a question…but it is a restriction on what will be accepted as evidence of a complete method.' The specification then considers a number of examples which centres should consider so that future candidates understand exactly what is required when this request appears in future series. This command phrase features in questions 3 and 4.

The word 'determine' in a question does not simply imply that candidates should find the answer but, to quote the specification, 'this command word indicates that justification should be given for any results found, including working where appropriate.' This command word features in question 7.

The phrase 'Show that' generally indicates that the answer has been given, and that candidates should provide an explanation that has sufficient detail to cover every step of their working. This command phrase features in questions 5, 6, 10 and 12(i)(b).

Whilst there is no specific level of working needed to justify answers to questions which use the command word 'find …', method marks may still be available for valid attempts that do not result in a correct answer, and standard advice (included in the specification) that candidates should state explicitly any expressions, integrals, parameters and variables that they use a calculator to evaluate (using correct mathematical notation rather than model specific calculator notation).

There are a number of examples where the question specifically asks for an exact value or in surd form, an approximate decimal equivalent will not gain full credit, examples are questions 5 and 6. Conversely question 10 specifies the level of accuracy to give the final answer. Regardless of the final required accuracy, candidates should be careful of not rounding prematurely, but also take care to avoid over-specifying rounded answers where the context does not support that level of accuracy.

Section A overview

The section A covers pure mathematics content of the H240 specification. Whilst not a major issue for this year's cohort, teachers using this (and subsequent papers) may want to remind candidates that the level of demand ramps up to the final few questions in section A, which are designed to be challenging and that the section B will start with less demanding questions before ramping up a second time. Candidates may benefit from leaving the more challenging section A questions until after answering the more straight forward mechanics questions in section B before returning to those missed questions.

Question 1 (i)

1 A circle with centre C has equation $x^2 + y^2 + 8x - 2y - 7 = 0$.

Find

(i) the coordinates of C, **[2]**

This proved to be a good start for nearly all candidates with the vast majority correctly completing the square (twice) to find the coordinates of the centre of the circle. When errors occurred these were nearly always down to sign errors inside the two brackets.

Question 1(ii)

(ii) the radius of the circle. **[1]**

Nearly all candidates stated the radius of the circle correctly in either part (i) or part (ii).

Question 2

2 Solve the equation $|2x-1| = |x+3|$. **[3]**

This question was answered extremely well with nearly all candidates correctly solving this equation involving the modulus function. Of the two main methods for solving this type of equation the first, which involved re-writing as $(2x-1)^2 = (x+3)^2$ was far more successful than those candidates who decided to re-write as two linear equation as many made sign errors even though most started from the correct two equations $(2x-1) = \pm(x+3)$.

Question 3

3 In this question you must show detailed reasoning.

A gardener is planning the design for a rectangular flower bed. The requirements are:

- the length of the flower bed is to be 3 m longer than the width,
- the length of the flower bed must be at least 14.5 m,
- the area of the flower bed must be less than 180 m^2.

The width of the flower bed is x m.

By writing down and solving appropriate inequalities in x, determine the set of possible values for the width of the flower bed. **[6]**

As this was a detailed reasoning question it was expected that candidates would do just that and show sufficient reasoning so that examiners could see that a complete analytical method had been employed. So it was therefore not possible to award full marks to those candidates who wrote statements such as $x(x+3)<180 \Rightarrow -15<x<12$. While many candidates correctly found that $11.5 \le x < 12$ a small proportion of candidates stated that $\frac{29}{6} \le x < \sqrt{60}$. This incorrect answer came from misreading the question and considering the length of the flower bed to be three times longer than its width (and not just 3 m longer than the width).

Question 4(i)(a)

4 In this question you must show detailed reasoning.

The functions f and g are defined for all real values of x by

$$f(x) = x^3 \quad \text{and} \quad g(x) = x^2 + 2.$$

(i) Write down expressions for

(a) $fg(x)$, **[1]**

Nearly all candidates correctly found the composite function fg(x) as $\left(x^2+2\right)^3$ although a number did expand the bracket in this part. Candidates are reminded that the number of marks available for a question or part-question are the best indicators to the amount of working and detail that is required.

Question 4(i)(b)

(b) $gf(x)$. **[1]**

Once again this was nearly always done correctly with the most common errors being those minority of candidates who stated that $\left(x^2\right)^3$ was equal to either x^5 or x^8.

Question 4(ii)

(ii) Hence find the values of x for which $\mathrm{fg}(x) - \mathrm{gf}(x) = 24$. **[6]**

There were a significant number of candidates that did not employ correct bracketing and mistakenly wrote $\left(x^2+2\right)^3 - x^6 + 2 = 24$ rather than $\left(x^2+2\right)^3 - \left(x^6+2\right) = 24 \Rightarrow \left(x^2+2\right)^3 - x^6 - 2 = 24$.

The majority of candidates went for expanding $\left(x^2+2\right)^3$ by writing out the bracket three times rather than using the binomial expansion of $(a+b)^n$.

While the majority rearranged their quartic into the form $6\left(x^2\right)^2 + 12x^2 - 18 = 0$ there were a number of candidates did not show sufficient working in solving this quartic, even though the full question required detailed reasoning.

Finally, it is expected at this level that as part of the detailed reasoning candidates should justify why $x^2 + 3 = 0$ did not provide any solutions.

Question 5(i)

5 **(i)** Use the trapezium rule, with two strips of equal width, to show that

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx \approx \frac{11}{4} - \sqrt{2}.$$ **[5]**

Candidates found this part extremely accessible and nearly all correctly derived the given result. Candidates are reminded though that in 'show that' questions suitable working must be shown and statements such as $\frac{2}{2}\left[\frac{1}{2} + \frac{1}{4} + 2\left(\frac{1}{2+\sqrt{2}}\right)\right] = \frac{11}{4} - \sqrt{2}$ are generally not acceptable.

Question 5(ii)

(ii) Use the substitution $x = u^2$ to find the exact value of

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx.$$ **[6]**

While nearly all candidates used the substitution correctly and re-wrote the integral as $\int_0^2 \frac{2u}{2+u}\,du$ many could not deal with the resulting improper fraction in the integrand. The most successful candidates re-wrote $\frac{2u}{2+u}$ either by using long division or realising that $\frac{2u}{2+u} = \frac{4+2u-4}{2+u} = 2 - \frac{4}{2+u}$. While examiners noted that some candidates employed more extreme methods (for example, further substitutions and the method of integration by parts) these were usually unsuccessful.

Question 5(iii)

(iii) Using your answers to parts **(i)** and **(ii)**, show that

$$\ln 2 \approx k + \frac{\sqrt{2}}{4},$$

where k is a rational number to be determined. **[2]**

While some, who had struggled with part (ii), left this part blank the majority of candidates equated their answers to parts (i) and (ii) with nearly all who were successful in part (ii) correctly determining that $k = \frac{5}{16}$.

Question 6(i)

6 It is given that the angle θ satisfies the equation $\sin\left(2\theta + \frac{1}{4}\pi\right) = 3\cos\left(2\theta + \frac{1}{4}\pi\right)$.

(i) Show that $\tan 2\theta = \frac{1}{2}$. **[3]**

Candidates were equally split in how to tackle this part. Approximately half expanding the brackets (using the correct compound-angle formulae) while the other half re-wrote as $\tan\left(2\theta + \frac{\pi}{4}\right) = 3$ before expanding. Both approaches proved equally successful in obtaining the expected result.

Question 6(ii)

(ii) Hence find, in surd form, the exact value of $\tan\theta$, given that θ is an obtuse angle. **[5]**

Many candidates did not read the question carefully and began their response by writing $2\theta = \tan^{-1}\left(\frac{1}{2}\right) \Rightarrow \theta = \ldots$ even though the question specifically asked for the exact value of $\tan\theta$. Of those candidates that used the correct double-angle formula for $\tan 2\theta$ many derived the correct three-term quadratic in tan with most correctly stating that $\tan\theta = -2 \pm \sqrt{5}$. However, a significant proportion ended their response here and did not go on to determine the exact value of $\tan\theta$ given that θ is an obtuse angle. A full solution needed the explicit realisation that since $-2 + \sqrt{5} > 0$, $\tan\theta = -2 + \sqrt{5}$ would not give an obtuse angle and therefore the only valid solution was $\tan\theta = -2 - \sqrt{5}$.

Question 7

7 The gradient of the curve $y = \mathrm{f}(x)$ is given by the differential equation

$$(2x-1)^3\frac{\mathrm{d}y}{\mathrm{d}x}+4y^2=0$$

and the curve passes through the point $(1,\ 1)$. By solving this differential equation show that

$$\mathrm{f}(x)=\frac{ax^2-ax+1}{bx^2-bx+1},$$

where a and b are integers to be determined. **[9]**

The responses to this final question in the pure section were mixed with examiners reporting a mixture of excellent responses followed by those that struggled with both the integration and the resulting algebraic manipulation required to obtain the answer in the required form. While most correctly separated the variables and wrote $-\frac{1}{4}\int\frac{\mathrm{d}y}{y^2}=\int\frac{\mathrm{d}x}{(2x-1)^3}$ many candidates had issues with the placement of the fraction on the left-hand side with examiners reporting that frequently this became a 4 rather than remaining as a quarter. While many candidates correctly integrated and remembered to include an arbitrary constant many decided to re-arrange their equation before attempting to find this constant; candidates are advised that in the majority of situations it is probably wisest to work out the +*c* immediately. Of those that obtained a correct particular solution to this differential equation, for example, $\frac{1}{y}=-\frac{1}{(2x-1)^2}+2$ many did not know the correct method for obtaining the result for f(x) in the required form. Many candidates took the reciprocal of each term separately rather than combining all relevant fractions first before taking the reciprocal and then expanding the brackets.

Section B overview

Two general points with regards to the answering of certain mechanics questions should be made in this overview.

The first is that unless told otherwise the value that candidates should use for the acceleration due to gravity, g, is 9.8 and not 10 or 9.81 (and this value is stated explicitly on the front cover of the examination paper).

Secondly, when applying Newton's second law in the context of connected particles, centres (when teaching) and candidates (when answering examination questions) are strongly encouraged to apply $F = ma$ to each particle separately rather than attempting to apply this equation to the whole system. These attempts are generally result in either the incorrect number of forces on the left-hand side of the equation or errors with the mass/acceleration of the combined system on the right-hand side. Often these attempts score no marks (as was commonly seen in this paper in question 12(iii)).

Question 8(i)

8 In this question $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ denote unit vectors which are horizontal and vertically upwards respectively.

A particle of mass 5 kg, initially at rest at the point with position vector $\begin{pmatrix}2\\45\end{pmatrix}$ m, is acted on by gravity and also by two forces $\begin{pmatrix}15\\-8\end{pmatrix}$ N and $\begin{pmatrix}-7\\-2\end{pmatrix}$ N.

(i) Find the acceleration vector of the particle. **[3]**

A significant number of candidates either forgot to include a gravity term or wrote that $\begin{pmatrix}15\\-8\end{pmatrix}+\begin{pmatrix}-7\\-2\end{pmatrix}+\begin{pmatrix}0\\-9.8\end{pmatrix}=5\mathbf{a}$ therefore forgetting to multiply the gravity term by the mass of the particle.

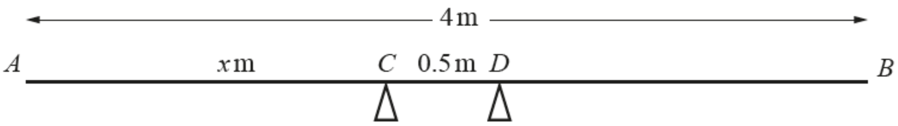
Question 8(ii)

(ii) Find the position vector of the particle after 10 seconds. **[3]**

The majority of candidates correctly applied the vector form of $\mathbf{s}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ in an attempt to find the position vector of the particle after 10 seconds, and it was generally only the errors mentioned in part (i) that stopped candidates from scoring full marks in this part.

Question 9(i)

9 A uniform plank AB has weight 100 N and length 4 m. The plank rests horizontally in equilibrium on two smooth supports C and D, where $AC = x$ m and $CD = 0.5$ m (see diagram).



The magnitude of the reaction of the support on the plank at C is 75 N. Modelling the plank as a rigid rod, find

(i) the magnitude of the reaction of the support on the plank at D, **[1]**

Nearly all candidates correctly stated that the magnitude of the reaction of the support on the plank at *D* was 25 N

Question 9(ii)

(ii) the value of x. **[3]**

The most common point on the plank to take moment about in this part was *A* and most who did so were successful in finding *x*. Of those that considered moments about a different point, while some were successful, too often examiners noted that candidates did not include all the required terms.

Question 9(iii)

A stone block, which is modelled as a particle, is now placed at the end of the plank at B and the plank is on the point of tilting about D.

(iii) Find the weight of the stone block. **[3]**

Many candidates struggled with this part and did not realise that if the plank was about to tilt about *D* then this was the point to take moments about as the reaction at *C* is therefore zero in this limiting case.

Candidates are strongly advised when taking moments to make it clear to the examiners which point (in this case on the plank) they are taking moments about. A number of candidates attempted to take moments about another point together with resolve forces vertically; these attempts were usually unsuccessful.

Key:

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Question 9(iv)(a)

(iv) Explain the limitation of modelling

(a) the stone block as a particle, **[1]**

It was pleasing that candidates were well prepared for this type of question with many correctly stating that the modelling the stone as a particle assumes that the weight of the block acts exactly at *B* and therefore the block's dimensions (or the distribution of the mass of the block) has not been taken into account. A number of candidates incorrectly stated that modelling the block as a particle implied that the block had negligible mass.

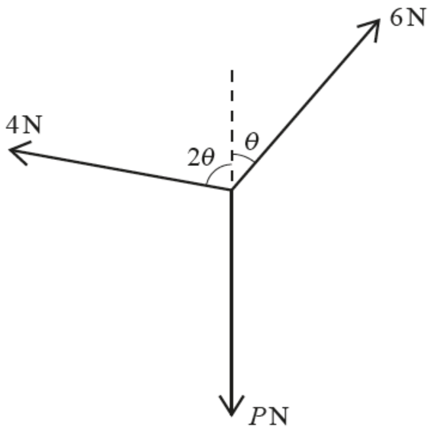
Question 9(iv)(b)

(b) the plank as a rigid rod. **[1]**

This part was also answered extremely well, with many correctly stating that modelling the block as a rigid rod assumes that the plank remains in a straight line and does not bend. A number of candidates, however, assumed that friction and/or air resistance had some relevance to this part.

Question 10(i)

10 Three forces, of magnitudes 4 N, 6 N and P N, act at a point in the directions shown in the diagram.



The forces are in equilibrium.

(i) Show that $\theta = 41.4°$, correct to 3 significant figures. **[4]**

For those candidates who correctly resolve horizontally and stated that $4\sin 2\theta = 6\sin\theta$ most went on to show with sufficient detail the required given answer. The most common errors, as expected, were those that did not deal correctly with the two different angles or those that had the standard confusion with sine and cosine when resolving forces.

Question 10(ii)

(ii) Hence find the value of P. **[2]**

This part was answered extremely well with the majority of candidates correctly resolving vertically and obtaining $P = 4\cos 2\theta + 6\cos\theta \Rightarrow P = 5$.

Question 10(iii)(a)

The force of magnitude 4 N is now removed and the force of magnitude 6 N is replaced by a force of magnitude 3 N acting in the same direction.

(iii) Find

(a) the magnitude of the resultant of the two remaining forces, **[3]**

While nearly all candidates understood the method for finding the magnitude of the resultant of the two remaining forces it was generally those candidates who resolved horizontally and vertically and then applied Pythagoras who were the more successful. Examiners noted that those candidates who attempted to form a triangle of forces and then use the cosine rule were generally not as successful making errors with the correct placement of the required lengths (representing the magnitudes) on the sides of the triangle.

Question 10(iii)(b)

(b) the direction of the resultant of the two remaining forces. **[2]**

When giving the direction of a force candidates are reminded that giving just the angle is insufficient. Candidates are strongly advised to draw a diagram (with an arrow on the resultant and/or the components) and to state that their angle is either 'below/above the horizontal' or 'to the upward/downward vertical'.

Question 11(i)

11 The velocity $v\,\mathrm{m\,s^{-1}}$ of a car at time t s, during the first 20 s of its journey, is given by $v = kt + 0.03t^2$, where k is a constant. When $t = 20$ the acceleration of the car is $1.3\,\mathrm{m\,s^{-2}}$. For $t > 20$ the car continues its journey with constant acceleration $1.3\,\mathrm{m\,s^{-2}}$ until its speed reaches $25\,\mathrm{m\,s^{-1}}$.

(i) Find the value of k. **[3]**

Nearly all candidates correctly differentiated the expression for v and correctly obtained the value of k as 0.1.

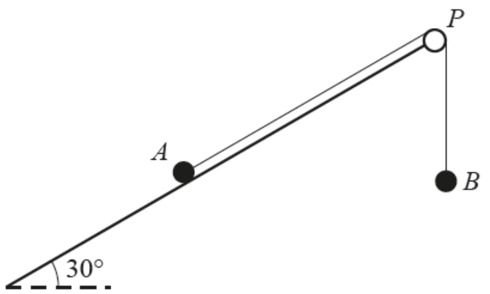
Question 11(ii)

(ii) Find the total distance the car has travelled when its speed reaches $25\,\text{m}\,\text{s}^{-1}$. **[7]**

This part was answered extremely well with many candidates correctly finding the total distance that car had travelled when its speed had reached $25\,\text{ms}^{-1}$. Many correctly realised that they had to use integration to find an expression for the displacement in terms of *t* which they could then use to find the distance travelled by the car in the first 20 seconds. However, many ignored the constant of integration that would arise from the corresponding indefinite integral; even though this constant was zero it is mathematically incorrect to simply ignore it (and for full marks candidates either had to consider this displacement expressed as a definite integral or explain why the constant was zero). Most candidates then used the SUVAT equations to work out the remaining distance travelled when the speed increased from 14 to 25 and correctly calculated the total distance as 265 m.

Question 12(i)(a)

12 One end of a light inextensible string is attached to a particle A of mass m kg. The other end of the string is attached to a second particle B of mass λm kg, where λ is a constant. Particle A is in contact with a rough plane inclined at 30° to the horizontal. The string is taut and passes over a small smooth pulley P at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane. The particle B hangs freely below P (see diagram).



The coefficient of friction between A and the plane is μ.

(i) It is given that A is on the point of moving down the plane.

(a) Find the exact value of μ when $\lambda = \frac{1}{4}$. **[7]**

This question was answered extremely well with most candidates setting out their working clearly and explaining what they were doing at each step. As mentioned in the overview candidates are strongly advised to deal with each particle separately and many correctly resolved vertically for *B*, and resolved both parallel and perpendicular to the plane for *A* using $F \le \mu R$ to find the value of μ when $\lambda = \frac{1}{4}$.

Question 12(i)(b)

(b) Show that the value of λ must be less than $\frac{1}{2}$. **[2]**

This part differentiated well with very few candidates showing clearly why $\lambda < \frac{1}{2}$. Only the most able realised that resolving parallel to the plane and then considering the fact that frictional force had to be positive was the easiest way of tackling this part. A number of candidates started by assuming that $\lambda = \frac{1}{2}$ and then concluding that the magnitude of the frictional force would therefore be zero. It was rarely felt by examiners that in these situations candidates made it clear why it was therefore less than (and not greater than) a half. In general, candidates are reminded that 'show that' questions are just that; the information in the question should be used to show a given result and therefore the result (or a limiting case of it) should not be used in an attempt to show that the given result is indeed true.

Question 12(ii)

(ii) Given instead that $\lambda = 2$ and that the acceleration of A is $\frac{1}{4}g\,\mathrm{m\,s^{-2}}$, find the exact value of μ. **[5]**

Like part (ii) this part differentiated well with many candidate unaware that if the value of λ was now greater than a half then particle *A* would move up the plane; many candidates assumed that the motion of *A* was down the plane. Of those that did have the correct direction of motion most correctly applied Newton's second law to both particles separately and obtained the correct value for the coefficient of friction.

Clear diagrams, with arrows showing the direction of motion and/or acceleration help to reduce the risk of sign errors when identifying the direction that any frictional force will act.

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