OCR
Oxford Cambridge and RSA

…day June 20XX – Morning/Afternoon

A Level Further Mathematics A

Y542 Statistics

SAMPLE MARK SCHEME

Duration: 1 hour 30 minutes

MAXIMUM MARK 75

This document consists of 16 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(i)		Independent and controlled	B1 [1]	1.2	Both, no others	
1	(ii)		$d = 1.61v - 24.1$	B2 [2]	1.1 3.3	All correct including letters, 3 s.f. BC **B1** Numbers right but not letters	$[d = 1.614v - 24.143]$
1	(iii)		$d = 1.61 \times 45 - 24.1 = 48$ to the nearest whole number	B1 [1]	3.4	awrt 48.5	
1	(iv)		Yes as r is close to 1 and 45 is within data range	E1 E1 [2]	3.5a 3.5b	Yes with one reason Second reason	
2	(i)		$\sum J + \sum K \sim \text{N}(12.4,\ 0.0344)$ $\text{P}(>12.7) = 1 - 0.9471 = 0.0529$	M1 A1 A1 [3]	1.1a 1.1 1.1	Consider the sum ~ N(12.4,... Standard deviation or variance correct awrt 0.053 BC	0.232 or 0.68: **M1A0**
2	(ii)		$K - 0.75J \sim \text{N}(0.05,\ 0.003625)$ $\text{P}(>0) = \Phi(0.08305) = 0.7969$	M1 A1 A1 [3]	1.1a 1.1 1.1	Or $4K - 3J \sim \text{N}(0.2, ...)$ Standard deviation or variance correct 0.0043 or 0.085: **M1A0** awrt 0.797 BC	

Question			Answer	Marks	AO	Guidance	
3	(i)		x (£): 1, 2, 3, 6, 10 $P(X=x)$: $\frac{1}{6}$, $\frac{2}{6}$, $\frac{1}{6}$, $\frac{1}{6}$, $\frac{1}{6}$	**M1** **A1** **A1** **[3]**	**3.1b** **1.1** **1.1**	x-values correct At least 2 probabilities correct All correct	
3	(ii)		$\sum xP(x) = \frac{1}{6}+\frac{4}{6}+\frac{3}{6}+\frac{6}{6}+\frac{10}{6} \quad = 4$	**B1**	**2.2a**	For dismissing the £5 loss, or using profit y: y (£): –4, –3, –2, 1, 5 $P(Y=y)$: $\frac{1}{6}$, $\frac{2}{6}$, $\frac{1}{6}$, $\frac{1}{6}$, $\frac{1}{6}$ giving $\sum yP(y) = -1$ and $\sum y^2P(y) = \frac{32}{3}$	
			$\sum x^2P(x) - \mu^2 = \frac{1}{6}+\frac{8}{6}+\frac{9}{6}+\frac{36}{6}+\frac{100}{6} - \mu^2$ $= \frac{77}{3} - \mu^2$ $= 9\frac{2}{3}$	**M1** **A1**	**1.1** **1.1**	Allow their value of μ	
			Therefore for 120 games the standard deviation is $\sqrt{120 \times 9\frac{2}{3}} = 34.1$	**M1** **A1FT** **[5]**	**2.2a** **1.1**	Multiply by 120 and take $\sqrt{\ }$ In range [34(.0), 35.1]	

Question			Answer	Marks	AO	Guidance	
4	(i)		<table><tr><td>7</td><td>2</td><td>1</td><td>17</td><td>11</td><td>14</td><td>4</td></tr><tr><td>4</td><td>2</td><td>1</td><td>7</td><td>6</td><td>5</td><td>3</td></tr><tr><td>+</td><td>–</td><td>–</td><td>+</td><td>–</td><td>+</td><td>+</td></tr></table>	M1	1.1	Calculate differences, rank them and attach signs	Follow through with correct signs and ranks from incorrect differences
			H_0 : population median difference $= 0$	B1	2.5	Hypotheses correctly stated	
			H_1 : population median difference $\neq 0$				
			$P = 4+7+5+3 = 19$				
			$Q = 1+2+6 = 9$	A1	3.3	*P* or *Q* correct	
			$T = 8$	A1	3.4	Both *P* and *Q* seen, *T* correct	**SC3**: Two-sample, max $3/6$
			$T_{crit} = 3;\ 8 > 3$	B1	1.1	Comparison with correct T_{crit}	
			Do not reject H_0. Insufficient evidence of a difference in test scores	A1FT	2.2b	Correct conclusion, in context, acknowledge uncertainty FT their *T* but not their T_{crit}	
				[6]			
4	(ii)		Uses magnitude of differences oe	B1 [1]	3.5b		
5	(i)		Goals are scored independently	B1	1.2	*Not* "singly"	
			Goals are scored at uniform rate	B1	1.2	Must be in context	Allow "constant average rate" but not "constant rate". **B0** for any answer that implies fixed numbers in given time. **B0** for "events must occur randomly", "independently", "singly" or "at constant rate" oe
				[2]			

Question			Answer	Marks	AO	Guidance	
5	(ii)	(a)	$P(X=r)=\mathrm{e}^{-1.9}\dfrac{1.9^r}{r!}$	B1 [1]	1.1	Must be seen	
5	(ii)	(b)	$P(X=3)=0.171$	B1 [1]	1.1		
5	(iii)		Total $\sim \mathrm{P_o}(1.9+\lambda)$ $\lambda=(1.9+1.31),\ \mathrm{P}(>3)=0.399...$ $\lambda=(1.9+1.32),\ \mathrm{P}(>3)=0.401...$ 0.399… < 0.4 and 0.401… > 0.4, hence a reasonable estimate is 0.4	M1 M1 A1 E1 [4]	2.2a 3.1b 1.1 3.2a	Use 1.9 + λ Evaluate RH tail probability for 1.31 and 1.32 Both evaluations correct	 BC BC
6	(i)	(a)	$X \sim \text{Geo}\left(\frac{1}{4}\right)$	B1 [1]	2.5	Accept Geo $\left(\frac{1}{4}\right)$ oe	
6	(i)	(b)	$\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^7$ $=\frac{4725}{16384}$	M1 A1 [2]	1.1 1.1	Expression of the form $\left(\frac{3}{4}\right)^a-\left(\frac{3}{4}\right)^b$ with $a < b$ awrt 0.288	Or $(1-q^7)-(1-q^3)$, $p(q^3+q^4+q^5+q^6)$
6	(ii)		$\mathrm{E}(X)=2 \Rightarrow p=\frac{1}{2}$ Hence $w = 6$	M1 A1 [2]	2.2a 2.2a		
6	(iii)		$\left(\frac{1}{2}\right)^4=\frac{1}{16}$	M1 A1 [2]	3.1a 1.1	Or, e.g. $\left(\frac{3}{4}\right)\times\left(\frac{3}{8}\right)^4\div\left(\frac{3}{4}\right)^5$	

Question			Answer	Marks	AO	Guidance	
7			$\mu = \bar{x} = 1.52$	**B1**	**3.1b**	1.52 seen	(α) Unless wrong working is seen, $p = 0.0808$ or $z = -1.4$ automatically gets **M1A1** and (unless hypotheses are given in terms of 1.52) automatically qualifies for **A1M1A1FT**
			$\hat{\sigma}^2 = \frac{49}{48}\left(\frac{120.8896}{49} - 1.52^2\right)$ $= 0.16$	**B1**	**3.3**	Biased estimate (0.1567) **B0** but can get all subsequent marks	
			$H_0 : \mu = 1.6$	**B1**	**1.1**	Hypotheses both correct, **B2**. One error, **B1**, but use of x or $\bar{x}$ or 1.52 is **B0B0**	(β) If neither $p = 0.0808$ or $z = -1.4$ is seen, all of the last 5 marks depend on seeing *either* $N\left(1.6, \frac{0.16}{49}\right)$ oe, *or* $\frac{1.52 - 1.6}{\sqrt{\frac{0.16}{49}}}$. Either of these seen but with square root errors can get **M1A0A1M1A1FT**
			$H_1 : \mu < 1.6$	**B1**	**2.5**		
			$p = 0.0808$ or $z = \frac{1.52 - 1.6}{\sqrt{\frac{0.16}{49}}} = -1.4$	**M1**	**2.1**	Evidence for 49 divisor needed *(see notes)*	
				A1	**3.4**	$p = 0.0808$ or $z = -1.4$ seen, allow $+1.4$ BC	
			$0.0808 > 0.05$ or $-1.4 > -1.645$	**A1**	**1.1**	Allow $1.4 < 1.645$ only if consistent	
			Do not reject H_0	**M1**	**1.1**	Correct method, comparison and conclusion	
			Insufficient evidence that height of plants using new plant food is less than 1.6	**A1FT**	**2.2b**	Contextualised, acknowledge uncertainty, needs double negative [*not* "evidence that height *is* 1.6"]. FT on *z*. Do not award final **M1A1** if *either* 49 divisor missing *or* hypotheses given in terms of 1.52	(γ) "cdfnorm" notation, or similar, with wrong *p* or *z* does *not* qualify for **M1A0A1** but can get last **M1A1FT** provided 49 is seen to be used and hypotheses not stated in terms of 1.52. "cdfnorm" notation with correct *p* or *z* can get full marks.
				[9]			

Question			Answer	Marks	AO	Guidance	
8	(i)		$\mu = \int_0^\infty 0.8x\mathrm{e}^{-0.8x}\mathrm{d}x = 1.25$	M1 A1	1.1a 1.1	Attempt $\int x\mathrm{f}(x)\,\mathrm{d}x$ Obtain 1.25 or exact equivalent	BC
			$\mathrm{E}(X^2) = \int_0^\infty 0.8x^2\mathrm{e}^{-0.8x}\mathrm{d}x\ [=3.125]$				
			$\mathrm{Var}(X) = 3.125 - 1.25^2$	M1	1.1	Attempt $\int x^2\mathrm{f}(x)\,\mathrm{d}x - \mu^2$	
			$= 1.5625$	A1 [4]	1.1	Obtain $\frac{25}{16}$ or exact equivalent	or awrt 1.56 BC
	(ii)		$\mathrm{P}(1 \le x < 2) = \int_1^2 0.8\mathrm{e}^{-0.8x}\mathrm{d}x$	M1	1.1	Correct pdf	
				E1	2.1	Integrate between 1 and 2	Requires clear use of notation
			$= 0.247432$ (6 s.f.)	A1	1.1	Correct answer, allow 3 s.f.	BC
			There are 60 specimens, so the expected frequency is $0.247432 \times 60 = 14.846$	E1 [4]	2.2a	Multiply probability by 60 and correctly obtain given answer AG	
	(iii)		H_0: data consistent with distribution	B1	2.5	Or equivalent	
			H_1: data not consistent				
			Combine cells to get	M1	1.1a	Combine last two cells	
			O | E | $(O-E)^2/E$ 24 | 33.040 | 2.4734 22 | 14.846 | 3.4474 10 | 6.6707 | 1.6613 4 | 5.4431 | 0.3826	M1	1.1	Calculate $\frac{(O-E)^2}{E}$ for at least one cell	
				A1	1.1	At least two $\frac{(O-E)^2}{E}$ values correct	
			$\sum \frac{(O-E)^2}{E} = 7.965$	A1	3.4	χ^2 in range [7.96, 7.97]	BC
			$\chi^2_3(0.95) = 7.815$ and $7.965 > 7.815$	B1	1.1	Comparison with 7.815	
			Reject H_0. Evidence that the data is not consistent with distribution	A1FT [7]	2.2b	State not consistent with distribution FT on numerical errors only	

Question			Answer	Marks	AO	Guidance	
9	(i)		$\mathrm{P}(Y \le y) = \mathrm{P}\left(\frac{1}{X^2} \le y\right)$	M1	1.1a	Attempt to write F_y in terms of X	
			$= \mathrm{P}\left(X \ge \frac{1}{\sqrt{y}}\right)$	E1	2.1	Make X the subject	
			$= 1 - \mathrm{F}\left(\frac{1}{\sqrt{y}}\right)$	M1	2.1	$1 - \mathrm{F}$ (inverse function)	
			$= \begin{cases} 1 - \frac{1}{16y} & y > \frac{1}{16}, \\ 0 & \text{otherwise.} \end{cases}$	E1	3.1a	$1 - \frac{1}{16y}$ correct, www	
				B1	1.1	0 and ranges correct (independent)	Withhold if extra range(s) given
				[5]			

Question			Answer	Marks	AO	Guidance	
9	(ii)		PDF of y is $\frac{1}{16y^2}$	**M1**	**3.1a**	Differentiate CDF to find PDF of Y	
			$\int_{\frac{1}{16}}^{\infty} \frac{y}{16y^2}\,\mathrm{d}y$	**M1**	**1.1**	Multiply by y and integrate, using their limits	
			$= \left[\frac{1}{16}\ln y\right]_{\frac{1}{16}}^{\infty}$	**A1**	**2.1**	Integration must be shown explicitly	
			and $\ln y$ is undefined as $y \to \infty$	**E1**	**3.2a**	Correctly justify given statement	**A0A0** For "calculator gives math error" or similar
				[4]			
		OR	PDF of x is $\frac{1}{8}x$	**M1**		Differentiate CDF to find PDF of X	
			$\mathrm{E}(Y) = \mathrm{E}\left(\frac{1}{X^2}\right)$				
			$= \int_0^4 \frac{1}{x^2}\frac{1}{8}x\,\mathrm{d}x$	**M1**		Integrate $\frac{1}{x^2} \times$ PDF , limits 0, 4	
			$= \left[\frac{1}{8}\ln x\right]_0^4$	**A1**		Integration must be shown explicitly	
			and $\ln x$ is undefined as $x \to 0$	**E1**		Correctly justify given statement	**A0A0** For "calculator gives math error" or similar
				[4]			

Assessment Objectives (AO) Grid

Question	AO1	AO2	AO3(PS)	AO3(M)	Total
1(i)	1				**1**
1(ii)	1			1	**2**
1(iii)				1	**1**
1(iv)				2	**2**
2(i)	3				**3**
2(ii)	3				**3**
3(i)	2		1		**3**
3(ii)	3	2			**5**
4(i)	2	2		2	**6**
4(ii)				1	**1**
5(i)	2				**2**
5(ii)(a)				1	**1**
5(ii)(b)	1				**1**
5(iii)	1	1	2		**4**
6(i)(a)		1			**1**
6(i)(b)	2				**2**
6(ii)		2			**2**
6(iii)	1		1		**2**
7	3	3	1	2	**9**
8(i)	4				**4**
8(ii)	2	2			**4**
8(iii)	4	2		1	**7**
9(i)	2	2	1		**5**
9(ii)	1	1	2		**4**
Totals	**38**	**18**	**8**	**11**	**75**

PS = Problem Solving
M = Modelling

Summary of Updates

Date	Version	Change
October 2019	2	Amendments to the front cover rubric instructions to candidates

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