

# AS Further Mathematics 8FM0

## Specimen Paper - Further Statistics 2 Mark Scheme

Qu	Scheme											Marks	AO
1	Plate	A	B	C	D	E	F	G	H	I	J	M1 A1	1.1b 1.1b
	Judge	7	3	2	1	8	10	4	6	5	9		
	Age	8	1	2	3	10	7	9	6	4	5		
	$\sum d^2 = 1+4+0+4+4+9+25+0+1+16 \quad [= 64]$											M1	1.1b
	$r_s = 1 - \frac{6 \times "64"}{10 \times 99}$											M1	1.1b
	$= 0.61212... = \text{awrt } \underline{0.612}$											A1	1.1b
												<b>(5 marks)</b>	
<b>Notes</b>													
1 <sup>st</sup> M1 an attempt to rank judge against actual ages													
1 <sup>st</sup> A1 both correct ranks													
2 <sup>nd</sup> M1 for an attempt to find $\sum d^2$ (some correct $d$ values found and sum attempted)													
3 <sup>rd</sup> M1 for using their $\sum d^2$ in formula for $r_s$ with $n = 10$													
2 <sup>nd</sup> A1 for awrt 0.612 or exact fraction e.g. $\frac{101}{165}$													

# AS Further Mathematics 8FM0

## Specimen Paper - Further Statistics 2 Mark Scheme

Qu	Scheme	Marks	AO
2(a)	e.g. $F'_c(x) = \frac{1}{6}(9 - 2x)$	M1	3.4
	For a cdf $F'(x)$ must be ...0 for all values of $x$ but $< 0$	A1	2.4
		(2)	
(b)	$k \int (x^2 - 2x + \frac{2}{3}) dx = 1 \Rightarrow k \left[ \frac{1}{3}x^3 - x^2 + \frac{2}{3}x \right]_{(2)}^{(5)} = 1$	M1	1.1b
	So $k \left[ \left( \frac{5^3}{3} - 5^2 + \frac{2}{3} \times 5 \right) - \left( \frac{2^3}{3} - 2^2 + \frac{2}{3} \times 2 \right) \right] = 1$	M1	1.1b
	$k = \frac{1}{20}$	A1	1.1b
		(3)	
(c)		B1 B1	1.1b 1.1b
		(2)	
(d)	$E(X) = \int_2^5 x f_D(x) dx = \frac{1}{20} \left[ \frac{x^4}{4} - \frac{2}{3}x^3 + \frac{1}{3}x^2 \right]_{(2)}^{(5)}$	M1	1.1b
	$= \frac{1}{20} \left[ \left( \frac{5^4}{4} - \frac{2}{3} \times 5^3 + \frac{5^2}{3} \right) - \left( \frac{2^4}{4} - \frac{2}{3} \times 2^3 + \frac{2^2}{3} \right) \right]$	M1	1.1b
	$= \underline{\underline{4.0625}}$	A1	1.1b
		(3)	
(e)	Sketch shows negative skew (with mode at 5)	B1ft	2.4
	Therefore expect that <u>mean &lt; median</u>	dB1ft	2.2b
		(2)	
		(12 marks)	
<b>Notes</b>			
(a)	M1 for attempting to differentiate the cdf or a suitable sketch showing a max in the range		
	A1 for a suitable explanation that shows the model is not appropriate.		
(b)	1 <sup>st</sup> M1 for attempting to integrate with some correct integration ( $x^n \rightarrow x^{n+1}$ ) and set equal to 1		
	2 <sup>nd</sup> M1 for some correct use of limits of 2 and 5		
	A1 for $k = \frac{1}{20}$		
(c)	1 <sup>st</sup> B1 for a curve (may look like a straight line) of positive gradient with $f(2) > 0$ and $f(5) > f(2)$		
	2 <sup>nd</sup> B1 for clear indication of 2 and 5 on x axis and $f(x) = 0$ elsewhere. [ $f(2) > 0$ and $f(5) > f(2)$ ]		

## AS Further Mathematics 8FM0

### Specimen Paper - Further Statistics 2 Mark Scheme

(d)	1 <sup>st</sup> M1 for attempting $\int x f_D(x) dx$ and some correct integration ( $x^n \rightarrow x^{n+1}$ )
	2 <sup>nd</sup> M1 for some correct use of limits of 2 and 5
	A1 for 4.0625 (accept 4.06 or 4.062 or 4.063)
(e)	1 <sup>st</sup> B1ft for a suitable reason based on skewness or correct calculation of median = 4.21(44.)
	2 <sup>nd</sup> dB1ft for stating that mean < median <b>with</b> some reference to part (c)
	NB A correct calculation of the median (sight of 4.21 or better) with no reference to the sketch will only score B1B0

# AS Further Mathematics 8FM0

## Specimen Paper - Further Statistics 2 Mark Scheme

Qu	Scheme	Marks	AO
3	Area = $X(7 - X)$	M1	3.4
	$E(\text{Area}) = E(7X) - E(X^2)$ <u>or</u> $7\int_0^7 \frac{1}{7}x \, dx - \int_0^7 \frac{1}{7}x^2 \, dx$	M1	2.1
	$E(X) = 3.5$ (from formula booklet or from integration)	B1	1.1b
	$E(X^2) = \text{Var}(X) + [E(X)]^2$ <u>or</u> $\int_0^7 \frac{1}{7}x^2 \, dx = \left[ \frac{x^3}{21} \right]_0^7$ $= \frac{49}{12} + [E(X)]^2$ [= 16.3333...]	M1	1.1b
	$E(\text{Area}) = 7 \times "3.5" - [4.08\dot{3} + "3.5"{}^2]$	A1ft	2.1
	$= 8.1666..$ awrt <b>8.17</b>	A1	1.1b
		(6 marks)	
	<b>Notes</b>		
	1 <sup>st</sup> M1 for using the model to form expression for the area		
	2 <sup>nd</sup> M1 for using E(area) and splitting the integral		
	B1 for $E(X) = 3.5$		
	3 <sup>rd</sup> M1 for use of a method for finding $E(X^2)$ : formula with $\text{Var}(X)$ and $E(X)$ or integration		
	1 <sup>st</sup> A1 for a correct numerical expression for E(area), ft their E(X)		
	2 <sup>nd</sup> A1 for a final answer of awrt 8.17		
	[Designated as a problem/extended solution]		

# AS Further Mathematics 8FM0

## Specimen Paper - Further Statistics 2 Mark Scheme

Qu	Scheme	Marks	AO
4(a)	$H_0 : \rho = 0 \quad H_1 : \rho < 0$	B1	2.5
	5% one-tail cv for $r$ is: $(-)$ 0.5494	M1	1.1b
	Significant result so there is evidence of a (-ve) correlation	A1	2.2b
		(3)	
(b)	Significant result suggests there is evidence of a linear relationship. So it is consistent.	B1	2.4
		(1)	
(c)	Need to choose model of the form: $h = a + bw$	M1	3.3
	Need $S_{hw}$ so use $(S_{hw})^2 = r^2 \times S_{hh} \times S_{ww}$	M1	2.1
	Since $r < 0$ need negative square root so $S_{hw} = -189.4749...$	A1	2.3
	$b = \frac{S_{hw}}{S_{ww}} = \frac{-189.4749...}{470.5} \quad [= -0.402709...]$	M1	3.4
	$a = 86.5 - (-0.402709...) \times 10.5 \quad [90.72845...]$	M1	1.1b
	So $h = 90.7 - 0.403w$	A1	1.1b
		(6)	
(d)	$5^b = 2.01...$ i.e. loss of approximately 2%	B1	3.4
		(1)	
(e)	RSS = $S_{hh} \times (1 - r^2)$ , = 122.1916... awrt <b>122</b>	M1, A1	1.1b (x2)
		(2)	
(f)	Eric's residual = $80 - (90.7 - 0.403 \times 10) = -6.67$ awrt <b>-6.7</b>	M1, A1	3.4, 1.1b
		(2)	
(g)(i)	Current model doesn't take account of hearing function before No "control group" to establish how hearing function changes for others <u>not</u> working in the factory	B1	3.5b
(ii)	Instead of using variable $h$ use a variable that measures <u>difference</u> in hearing function since working in factory	B1	2.4
		(2)	
		<b>(17 marks)</b>	
<b>Notes</b>			
(a)	B1 for both hypotheses correct in terms of $\rho$		
	M1 for use of tables to find the cv of (condone $\pm$ )		
	A1 for a correct conclusion based on comparing $-0.620$ with $-0.5494$		
(b)	B1 for mention of the result being significant and saying it is consistent		
(c)	1 <sup>st</sup> M1 for selecting the appropriate model		
	2 <sup>nd</sup> M1 for a suitable strategy/formula to find $S_{hw}$		
	1 <sup>st</sup> A1 for $S_{hw} =$ awrt $-189$		
	3 <sup>rd</sup> M1 for a correct expression for $b$ (ft their $S_{hw}$ )		
	4 <sup>th</sup> M1 for a correct expression for $a$ (ft their $b$ )		

## AS Further Mathematics 8FM0

### Specimen Paper - Further Statistics 2 Mark Scheme

	2 <sup>nd</sup> A1 for a correct equation with $a = \text{awrt } 90.7$ and $b = \text{awrt } -0.403$
(e)	M1 for use of a correct formula (some correct substitution seen) A1 for awrt 122
(f)	M1 for a correct calculation using their line from (c) (allow $\pm$ ) A1 for awrt $-6.7$
(g)(i)	B1 for a suitable comment illustrating limitation of the current model
(ii)	B1 for suggesting change of variable $h$ to use idea of <u>difference</u> in hearing function