AS Further Mathematics 8FM0

Specimen Paper - Further Statistics 1 Mark Scheme

Question	Scheme	Marks	AOs
1(a)	Degrees of freedom $(2-1)(2-1) \rightarrow \chi^2_{(1,0.05)} = 3.841$	M1	3.1b
	[0.245 < 3.841, therefore do not reject H_0] There is not enough evidence to suggest an association between age and time spent on social media.	A1	2.2b
		(2)	
(b)	Some of the expected frequencies may fall below 5	B1	3.5b
		(1)	
(c)	$\sum \frac{(O-E)^2}{E} = 0.47 + 0.40 + ... + \frac{(7-\frac{92\times26}{200})^2}{\frac{92\times26}{200}} + \frac{(19-\frac{108\times26}{200})^2}{\frac{108\times26}{200}}$	M1	2.1
	awrt **<u>9.57</u>**	A1	1.1b
	$\nu = (5-1)(2-1) \rightarrow \chi^2_{(4,0.05)} = 9.488$	B1	3.1b
	[9.57 > 9.488, therefore reject H_0] There is an association between age and time spent on social media.	A1	2.2b
		(4)	
(d)	Part (c) uses more information, so should be more reliable.	B1	3.2b
		(1)	
			(8 marks)

Notes	
(a)	M1 Finding the critical value for the test A1 Correct conclusion in context
(b)	B1 Understanding that to carry out the test, expected frequencies should be > 5
(c)	M1 Calculating the test statistic A1 awrt 9.57 B1 Obtaining correct degrees of freedom and critical value for the test A1 Correct contextual conclusion
(d)	B1 Correct choice of conclusion and supporting reason

Question	Scheme	Marks	AOs
2(a)	$[\mathrm{E}(Y)=]2^2\times\frac{1}{50}\ +\ 3^2\times a\ +\ 6^2\times\frac{1}{25}\ +\ 11^2\times b=50.3$ $\frac{1}{50}+a+\frac{1}{25}+b=1$	M1	3.1a
	$\left.\begin{matrix}9a+121b=48.78\\ a+b=0.94\end{matrix}\right\}\rightarrow 112b=40.32$	M1	1.1b
	$\underline{a=0.58}$ and $\underline{b=0.36}$	A1	1.1b
		(3)	
(b)	$\mathrm{P}(9-Y>0)\ [=\mathrm{P}(9-X^2>0]=\mathrm{P}(X<3)$	M1	1.1b
	$=\mathrm{P}(X=2)$ $\underline{\frac{1}{50}}$	A1	1.1b
		(2)	
(c)(i)	$[\mathrm{E}(T)=120\times(\frac{1}{50}+\frac{1}{25})=]$ **<u>7.2</u>**	B1	1.1b
		(1)	
(ii)	$[\mathrm{Var}(T)=120\times(\frac{1}{50}+\frac{1}{25})\times(1-(\frac{1}{50}+\frac{1}{25}))=]$ awrt **<u>6.77</u>**	B1	1.1b
		(1)	
(d)	$T\sim\mathrm{B}(120,\frac{3}{50})\rightarrow\mathrm{Po}(7.2)$	M1	3.3
	$\mathrm{P}(T>10)=1-\mathrm{P}(T\leq 10)\ [=1-0.88667\ldots]$	M1	1.1b
	awrt **<u>0.113</u>**	A1	1.1b
		(3)	
			(10 marks)

Notes	
(a)	1st M1 Realising that both equations are required 2nd M1 Solving simultaneously to eliminate one variable A1 Both $a = 0.58$ and $b = 0.36$
(b)	M1 For identifying $\mathrm{P}([-3<]X<3)$ or $\mathrm{P}(Y<9)$
(d)	1st M1 Selecting Poisson approximation to binomial 2nd M1 Using Poisson model A1 awrt 0.113 NB: Using exact binomial $\mathrm{P}(T>10)=0.10661...$

Question	Scheme	Marks	AOs
3(a)	$r = 80 \times P(X = 2)$ $s = 80 - (6.56 + 17.11 + \ldots)$ or $s = 80 \times (1 - P(X \leq 4))$	M1	3.4
	r = awrt **21.57** and s = awrt **6.99**	A1	1.1b
		(2)	
(b)	H_0: B(30, 0.08) is a suitable model H_1: B(30, 0.08) is not a suitable model	B1	2.5
	$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(14-6.56)^2}{6.56} + \ldots + \frac{(5-'6.99')^2}{6.99}$	M1	2.1
	= awrt **11.5**	A1	1.1b
	Degrees of freedom = 6 – 1 = 5	B1	1.1b
	$11.5 > \chi^2_{5,(0.05)} = 11.070$ so reject H_0	M1	1.1b
	B(30, 0.08) is not a suitable model for the number of empty hotel rooms.	A1	3.5a
		(6)	
(c)	Using the data to estimate p $\frac{0\times14+1\times18+2\times22+3\times11+4\times10+5\times5}{80} = 2$	M1	3.3
	B(30, $\frac{2}{30}$)	A1	1.1b
		(2)	
			(10 marks)

Notes	
(a)	M1 Either correct calculation A1 both r = awrt **21.57** and s = awrt **6.99**
(b)	B1 Both hypotheses 1st M1 Attempting to find the test statistic using $\chi^2 = \sum \frac{(O-E)^2}{E}$ 1st A1 awrt 11.5 B1 for df = 5 2nd M1 comparing test statistic with critical value 2nd A1 correct conclusion in context
(c)	M1 Suggesting using the data to improve the model A1 Correct model (also allow Po(2))

Question	Scheme	Marks	AOs
4(a)	[$X \sim$ Po(2.4) $Y \sim$ Po(1.6)] P(X = 2) = $\frac{e^{-2.4} \times 2.4^2}{2} = 0.261\ldots$ P(Y = 2) = $\frac{e^{-1.6} \times 1.6^2}{2} = 0.258\ldots$	M1	1.1b
	Therefore the photocopier is more likely to break down exactly twice.	A1	1.1b
		(2)	
(b)	P($X \leq 1$) × P($Y \leq 1$)	M1	1.1b
	[= 0.3084… × 0.5249…] awrt **<u>0.162</u>**	A1	1.1b
		(2)	
(c)	$\frac{P(X=4) \times P(Y=0) + P(X=3) \times P(Y=1)}{P(X+Y=4)}$	M1	3.1b
	$= \frac{\frac{e^{-2.4}2.4^4}{4!} \times e^{-1.6} + \frac{e^{-2.4}2.4^3}{3!} \times \frac{e^{-1.6}1.6}{1}}{\frac{e^{-4}4^4}{4!}}$	M1 M1	1.1b 1.1b
	$= \frac{297}{625}$ awrt **<u>0.475</u>**	A1	1.1b
		(4)	
(d)	H_0: $\lambda = 4$ $\mu = 8$ H_1: $\lambda < 4$ $\mu < 8$	B1	2.5
	$T \sim$ Po(8) P($T \leq 3$) [= awrt 0.0424]	M1	3.3
	[0.0424 < 0.05] Reject H_0	M1	1.1b
	There is evidence that the rate of breakdowns has decreased following the repairs.	A1	1.1b
		(4)	
			(12 marks)

Notes	
(a)	M1 Using each Poisson model to attempt each probability A1 Both correct awrt 3sf and correct conclusion
(b)	M1 Multiplication of two correct cumulative probabilities A1 awrt 0.162
(c)	1st M1 Correct ratio expression 2nd M1 Use of $P(X=4) \times P(Y=0) + P(X=3) \times P(Y=1)$ [0.125… × 0.201… + 0.209… × 0.323…] 3rd M1 Ratio of probabilities with denominator $\frac{e^{-4}4^4}{4!}$ [0.195…] A1 awrt 0.475
(d)	B1 Both hypotheses with λ or μ 1st M1 Using Po(8) to calculate P($T \leq 3$) 2nd M1 Correct non-contextual conclusion A1 Conclusion in context and no errors