

Pearson Edexcel Level 3

GCE Further Mathematics

Advanced Subsidiary

Further Pure Mathematics 2

Specimen paper

Time: 50 minutes

Paper Reference(s)

8FM0/22

You must have:

Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 4 questions in this section of the paper. The total mark is 40.
- The marks for each question are shown in brackets - *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Answer ALL questions. Write your answers in the spaces provided.

1. (i) The set $G = \{1, 3, 4, 9, 10, 12\}$ forms a group under the operation of multiplication modulo 13.
- (a) Copy and complete the Cayley table below, Table 1, for this group. (3)
- (b) Find a subgroup of (G, \times_{13}) of order 3. (1)
- (c) Explain why there can be no subgroup of (G, \times_{13}) of order 4. (1)
- (ii) Determine whether the set $\{1, 3, 6, 9, 12\}$ under the operation of multiplication modulo 15 forms a group. (3)

\times_{13}	1	3	4	9	10	12
1		3				
3			12			
4						
9			10			
10	10					
12					3	

Table 1

(Total for Question 1 is 8 marks)

2. The two sequences x_n and y_n satisfy the recurrence relations

$$\begin{aligned} x_1 &= 2 & x_{n+1} &= 5x_n - y_n \\ y_1 &= 1 & y_{n+1} &= 3x_n + y_n \end{aligned} \quad n \geq 1.$$

(a) Find the value of x_3 .

(2)

These recurrence relations can be written in matrix form as

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \mathbf{A} \begin{pmatrix} x_n \\ y_n \end{pmatrix}, \quad n \geq 1,$$

where $\mathbf{A} = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}$.

(b) Find

(i) the eigenvalues of \mathbf{A} ,

(ii) the corresponding eigenvectors of \mathbf{A} .

(5)

(c) Hence write down matrices \mathbf{P} and \mathbf{D} such that $\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \mathbf{D}$, where \mathbf{D} is a diagonal matrix.

(2)

New sequences u_n and v_n can be formed from x_n and y_n using the transformation

$$\begin{pmatrix} u_n \\ v_n \end{pmatrix} = \mathbf{P}^{-1} \begin{pmatrix} x_n \\ y_n \end{pmatrix}.$$

(d) Show that $\begin{pmatrix} u_{n+1} \\ v_{n+1} \end{pmatrix} = \mathbf{D} \begin{pmatrix} u_n \\ v_n \end{pmatrix}$.

(2)

(e) Hence find closed form expressions for the original sequences x_n and y_n .

(2)

(Total for Question 2 is 16 marks)

3. (a) Sketch, on an Argand diagram, the arc given by the locus of points z in the complex plane satisfying

$$\arg\left(\frac{z}{z-25i}\right) = \frac{11\pi}{12}.$$

(2)

The centre of the circle containing this arc is $x + yi$ where

$$x = -\frac{25}{2}(2 + \sqrt{3}).$$

- (b) Find the radius of this circle, giving your answer to 3 decimal places. **(4)**

In a game, players take turns to roll balls along a horizontal surface from a fixed starting position to a fixed target point 25 metres away.

The game can be modelled in the complex plane, with the starting position at the origin and the target at the point $25i$, where the units are metres.

In a particular game, the first player's ball has come to rest with its centre at $0.5 + 23.25i$. The second player's ball is following the path sketched in part (a).

Given that the radius of each ball is approximately 5 centimetres,

- (c) use this model to determine whether the second player's ball will hit the first player's ball. **(4)**

(Total for Question 4 is 10 marks)

4. A positive integer n satisfies the inequality $99 < n < 1000$.

The integer n is written as a three digit number abc such that

$$n = 100a + 10b + c,$$

where the digits a , b , and c are each integers between 0 and 9 inclusive, $a \neq 0$.

The integer n has the following three properties:

- n is divisible by 11,
- the sum of the digits of n is odd,
- $n \equiv 5 \pmod{9}$.

Use this information to find all possible values for n , making your reasoning clear.

(Total for Question 4 is 6 marks)

TOTAL FOR PAPER IS 40 MARKS