9FM0/4B: Further Statistics 02 Mark Scheme

Qu	Scheme	Marks	Grade	AO
1(a)	<table><tr><td>**Art**</td><td>*A*</td><td>*B*</td><td>*C*</td><td>*D*</td><td>*E*</td><td>*F*</td><td>*G*</td><td>*H*</td><td>*I*</td><td>*J*</td></tr><tr><td>**Rnk**</td><td>3</td><td>10</td><td>1</td><td>9</td><td>8</td><td>2</td><td>6</td><td>7</td><td>4</td><td>5</td></tr></table>			
	$\sum d^2 = 1 + 16 + 0 + 1 + 0 + 0 + 1 + 4 + 1 + 16$ [= 40]	M1	Low	1.1b
	$r_s = 1 - \frac{6\times"40"}{10(10^2-1)}$	M1	Low	1.1b
	= 0.7575…. awrt **<u>0.758</u>**	A1	Low	1.1b
		(4)		
(b)	$H_0 : \rho_s = 0$ $H_1 : \rho_s > 0$	B1	Med	2.5
	5% critical value is 0.5636	B1	Med	1.1a
	Reject H_0 there is evidence to support the editor's belief	B1	Med	2.2b
		(3)		
		(7 marks)		

	Notes
(a)	1st M1 for ranking the no of words 2nd M1 for an attempt at $\sum d^2$ 3rd M1 for use of their $\sum d^2$ in a correct formula A1 for awrt 0.758 or any exact equivalent
(b)	1st B1 for both hypotheses in terms of ρ_s or ρ with H_1 compatible with their ranking and r_s 2nd B1 for the correct cv of 0.5636 3rd B1 for a correct conclusion based on their cv and their r_s that mentions the editor's belief

9FM0/4B: Further Statistics 02 Mark Scheme

Qu	Scheme	Marks	Grade	AO
2(a)	[Let d = spatial skill – numerical skill] $\bar{d} = 4.875$	B1	Low	2.1
	$s_d = 5.27629\ldots$	B1	Low	1.1b
	$t_7(5\%)$ 2-tail cv = 2.365	B1	Low	1.1b
	95% CI for μ_d is: $4.875 \pm 2.365 \times \frac{5.27...}{\sqrt{8}}$	M1	Med	3.4
	= awrt **(0.46, 9.29)**	A1	Med	1.1b
		(5)		
(b)	0 is not in the interval	B1	Med	2.4
	Therefore does support the manager's belief	dB1	High	2.2b
		(2)		
(c)	The scores are not independent since relate to the same recruit.	B1	High	2.4
		(1)		
		(8 marks)		

	Notes
(a)	1st B1 for mean = awrt 4.87 or 4.88 (+) 2nd B1 for s_d = awrt 5.28
	M1 for use of correct formula, ft their mean, s_d and cv for t (use of 1.96 is M0)
	A1 for awrt (0.46, 9.29) or awrt (– 9.29, – 0.46)
(b)	1st B1 for correct statement about 0 and interval 2nd dB1 dep on 1st B1 for correct conclusion
(c)	B1 for a suitable reason that states or implies lack of independence since same recruits used

9FM0/4B: Further Statistics 02 Mark Scheme

Qu	Scheme	Marks	Grade	AO
3(a)	$\alpha < \dfrac{7s^2}{\sigma^2} < \beta$ where α and β are values from χ^2_7	M1	Med	3.3
	$\alpha = 2.167$ and $\beta = 14.067$	B1 B1	Med Med	1.1b 1.1b
	$s^2 = \dfrac{73094 - 8\times\left(\dfrac{752}{8}\right)^2}{7}$ [$= 343.71428\ldots$]	M1	Med	1.1b
	So $171.0386\ldots < \sigma^2 < 1110.2907\ldots$	M1	Med	1.1b
	Therefore **$\underline{13.1 < \sigma < 33.3}$**	A1 (6)	High	1.1b
(b)	The weights of apples from the tree are <u>normally distributed</u>	B1 (1)	Med	2.3
(c)	26g is in the confidence interval	B1	Low	2.4
	So this <u>does</u> support the asssertion	dB1 (2)	Med	2.2b
		(9 marks)		

	Notes
(a)	1st M1 for using the correct χ^2 model 1st B1 for one correct value from tables, 2nd B1 for both correct values 2nd M1 for a correct expression for s^2 3rd M1 for rearranging to obtain an interval for σ^2 A1 for awrt 13.1 and awrt 33.3 in a correct interval for σ
(b)	B1 for mentioning only valid if weights of apples follow a normal distribution
(c)	1st B1 for a reason that 26g is in CI 2nd dB1 dep on 1st B1 for a statement that this does support the assertion.

9FM0/4B: Further Statistics 02 Mark Scheme

Qu	Scheme	Marks	Grade	AO
4(a)	$\bar{x} = 33.16...$ awrt **33.2**	B1	Low	1.1b
	$s^2 = 37.77$ awrt **37.8**	B1	Low	1.1b
		(2)		
(b)	$H_0 : \sigma_b^{\ 2} = \sigma_g^{\ 2}$ $H_1 : \sigma_b^{\ 2} \neq \sigma_g^{\ 2}$	B1	Low	2.5
	$F_{8,5} = \frac{37.77}{15.77}$ [=2.39505…]	M1	Med	3.4
	$F_{8,5}(5\%\text{ two-tail})$ c.v. = 4.82	B1	Low	1.1b
	Not significant, insufficient evidence of a difference in variances	A1	Med	2.2b
		(4)		
(c)	$H_0 : \mu_g - \mu_b = 2$ $H_1 : \mu_g - \mu_b > 2$	B1	Med	2.1
	$s_p^{\ 2} = \frac{8\times37.77 + 5\times15.77}{9+6-2}$ [= awrt 29.31]	M1	Med	3.1b
	$t_{13} = \frac{(33.16.. - 27.45) - 2}{\sqrt{"29.31"\left(\frac{1}{9}+\frac{1}{6}\right)}}$ = , 1.302… awrt 1.30	M1,	Low	1.1b
		A1	Med	1.1b
	$t_{13}(5\%)$ one-tail cv = 1.771 [or p = 0.1076…]	B1	Low	1.1b
	Not significant, no support for the teacher's belief	A1	Med	2.2b
		(6)		
(d)	The times must be normally distributed	B1	Med	1.2
		(1)		
(e)	Test in (c) requires $\sigma_b^{\ 2} = \sigma_g^{\ 2}$, test in (b) suggests this is OK	B1	High	2.4
		(1)		
		(14 marks)		

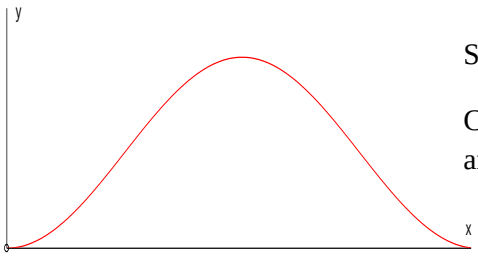
	Notes
(a)	1st B1 for awrt 33.2 and 2nd B1 for awrt 37.8 (found on their calculator)
(b)	1st B1 for both hypotheses in terms of σ. M1 for a correct calculation of the test statistic 2nd B1 for correct cv awrt 4.82 A1 for a correct conclusion accepting H_0
(c)	1st B1 for setting up suitable hypotheses. Both correct and in terms of μ. 1st M1 for correct expression for $s_p^{\ 2}$ 2nd M1 for correct expression for test staitistic 1st A1 for awrt 1.30 2nd B1 for a correct cv or p-value 2nd A1 for a correct conclusion mentioning the teachers belief (o.e.)
(d)	B1 for stating "normal"
(e)	B1 for correctly relating the test in (b) to the test in (c)

9FM0/4B: Further Statistics 02 Mark Scheme

Qu	Scheme	Marks	Grade	AO
5	[Let B_i be the length of bolt *i* and $\bar{B}_n$ = mean length of *n* bolts]			
	{2} $\mathrm{P}\left(\bar{B}_2 - \bar{B}_3 > \frac{2}{10}\right)$ <u>or</u> $\{2\}\mathrm{P}\left(\frac{B_1+B_2}{2} - \frac{B_1+B_2+B_3}{3} > \frac{2}{10}\right)$ (o.e.)	M1	Med	3.1b
	$\{2\}\mathrm{P}(3B_1+3B_2-2B_1-2B_2-2B_3>1.2)$	M1	High	2.1
	$\{2\}\mathrm{P}(B_1+B_2-2B_3>1.2)$	A1	High	1.1b
	Let $X = B_1+B_2-2B_3$ then $X \sim \mathrm{N}(0, 6\times0.3^2)$ or $X \sim \mathrm{N}(0, \sqrt{0.54}^2)$	M1 A1	High High	3.3 1.1b
	P($X > 1.2$) = 0.0512352…	M1	High	3.4
	So probability of difference is 2x 0.05123… = awrt **<u>0.102</u>**	A1cao	High	3.2b
		(7)		
		(7 marks)		

	Notes
	1st M1 for writing an appropriate probability (condone missing x 2) to start the problem 2nd M1 for a correct 1st step in particular realising can't use $X = \bar{B}_2 - \bar{B}_3$ (condone slip on 1.2) 1st A1 for a correct probability statement (condone missing x2) 3rd M1 for setting up an appropriate model for X ($X \sim \mathrm{N}(0, \ldots)$) 2nd A1 for a correct variance for their model 4th M1 for using the model (and calculator) to find an appropriate probability (ft slip on "1.2") 3rd A1cao for understanding the need for x2 to find prob of a <u>difference</u> [(ii) is an extended problem and a 3.1, 3.2 question]

9FM0/4B: Further Statistics 02 Mark Scheme

Qu	Scheme	Marks	Grade	AO
6(a)	 Symmetry and f(0) = f(2π)= 0	B1	Low	1.1b
	Correct shape with inflections and f(x) never negative	B1	Med	1.1b
		(2)		
(b)	$\mathrm{E}\left(X^2\right)=\frac{1}{2\pi}\int_0^{2\pi}(x^2-x^2\cos x)\,\mathrm{d}x=\frac{1}{2\pi}\left[\frac{x^3}{3}\right]_0^{2\pi}-\frac{1}{2\pi}\int_0^{2\pi}x^2\cos x\,\mathrm{d}x$	M1	Low	2.1
	$\int_0^{2\pi}x^2\cos x\,\mathrm{d}x=\left[x^2\sin x\right]_0^{2\pi}-2\int_0^{2\pi}x\sin x\,\mathrm{d}x$	M1	Med	2.1
	$=(0)-(0)-2\left\{\left[-x\cos x\right]_0^{2\pi}--\int_0^{2\pi}\cos x\,\mathrm{d}x\right\}$	M1	High	1.1b
	$\therefore \mathrm{E}\left(X^2\right)=\frac{4\pi^2}{3}-2$ **(*)**	A1cso	High	1.1b
		(4)		
(c)	Var(X) = $\mathrm{E}(X^2)-\pi^2$ [= 1.28986…]	M1	Med	1.1b
	So standard deviation = $\sqrt{1.28986...}$ = 1.13572… awrt **1.14**	A1	Med	1.1b
		(2)		
(d)	IQR of X is $2\times(\pi-2.31)=1.663...$ awrt 1.66	B1	Med	3.1a
	IQR using normal is $2\times z\times$"1.1357..." (= 1.532…)	M1 A1ft	Med Med	2.1 1.1b
	So percentage error is $\frac{\text{"1.663..."}-\text{"1.532..."}}{\text{"1.663"}}\times 100$ = awrt **8%**	A1	Med	1.1b
		(4)		
		(12 marks)		

	Notes
(a)	1st B1 for symmetric shape and touching/crossing axis at 0 and 2π 2nd B1 for "bell shape" and no curve below x axis
(b)	1st M1 for a correct expression and some correct integration of x^2 term (ignore limits) 2nd M1 for an attempt at integration by parts 1st stage 3rd M1 for 2nd integration by parts and use of correct limits
(*)	A1 cso for a fully correct solution…no incorrect signs seen and all Ms scored
(c)	M1 for Use of E(X) = π and correct expression for Var(X) A1 for awrt 1.14
(d)	B1 for IQR = awrt 1.66 or a correct expression M1 for a correct method to find IQR using a normal where 0.67 ,, z ,, 0.68 and ft their sd 1st A1ft for a correct expression 0.67 ,, z ,, 0.68 and ft their sd A1 for awrt 8%

	[(b) involves work from pure for the integration]

9FM0/4B: Further Statistics 02 Mark Scheme

Qu	Scheme	Marks	Grade	AO
7. (a)	$r = \dfrac{2076.8}{\sqrt{246 \times 20\,461.16}}$	M1	Low	1.1b
	$= 0.92568\ldots$ awrt **<u>0.926</u>**	A1 (2)	Low	1.1b
(b)	$H_0 : \rho = 0 \quad H_1 : \rho > 0$	B1	Low	2.1
	5% cv for r is 0.6215	B1	Low	1.1b
	Significant, there is evidence of a positive correlation between m and t	B1 (3)	Med	2.2b
(c)(i)	(b) says there is positive correlation so can use a linear model	B1	High	2.4
(ii)	To estimate turnover need $t = a + bm$	M1	High	3.3
	$b = \dfrac{2076.8}{246} = 8.442\ldots$ awrt **<u>8.44</u>**	A1	Med	1.1b
	$a = 62 - 8b = -5.538..$ awrt **<u>– 5.54</u>**	A1 (4)	Med	1.1b
(d)	Increase is $3b$	M1	Med	3.4
	$= 25.32$ or **<u>£ 253 200</u>**	A1 (2)	High	1.1b
(e)	Suitable sketch (use overlay)	B2/1/0 (2)	Low/Med	1.1b(x2)
(f)	Average of residuals for 14 and 18 $\approx$ 15 or 16	M1	High	3.4
	So estimate of 25.3 + 15 or 16 $\approx$40 so **<u>£400 000</u>**	A1 (2)	High	1.1b
(g)	$\dfrac{p^4 \times q^4}{8C4\,p^4 \times q^4} = \dfrac{1}{70}$	M1	High	3.1a
		A1 (2)	High	1.1b
(h)	e.g. collect more data	B1 (1)	High	2.4
		(18 marks)		

	Notes
(a)	M1 for a correct expression A1 for awrt 0.926
(b)	1st B1 for both hypotheses in terms of ρ 2nd B1 for the correct cv of 0.6215 3rd B1 for a correct conclusion based on their cv and their r
(c)(i)	B1 for suitable reason for linear model
(ii)	M1 for choosing the correct linear model 1st A1 for b = awrt 8.44 and 2nd A1 for a = awrt – 5.54 (or A1A1 for $t = 8.44e - 5.54$)
(d)	M1 for attempting $3b$ (o.e.) or sight of 25.32 A1 for awrt £ 253 000
(e)	B2/1/0 for suitable sketch (– 1 eeoo)
(f)	M1 for a suitable calculation based on $\approx$ 25 (ft (d)) + something greater than 10 A1 provided M1 scored for an answer in the range 350 000 ~ 450 000

(g)	M1 for selecting a suitable probability model (conditional with any value for p and $q = 1 - p$)
(h)	B1 for a sensible suggestion