Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Sample Assessment Material

(Time: 1 hour 30 minutes) | Paper Reference **9FM0/3A**

Further Mathematics

Advanced

Paper 3A: Further Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. The hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Show that an equation of the tangent to H at the point $P\,(a\sec\theta, b\tan\theta)$ is

$$ya\tan\theta = xb\sec\theta - ab$$

(4)

Question 1 continued

(Total for Question 1 is 4 marks)

2. Relative to a fixed origin O, the points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively, where

$$\mathbf{a} = \mathbf{i} + \mathbf{j} \qquad \mathbf{b} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} \qquad \mathbf{c} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$$

Find

(a) the area of triangle OBC

(2)

(b) the volume of the tetrahedron $OABC$

(2)

Question 2 continued

(Total for Question 2 is 4 marks)

3. (a) Using the substitution $t = \tan\left(\frac{x}{2}\right)$ show that the equation

$$4\tan x + 3\cot\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right) = 0$$

can be written as

$$3t^4 - 8t^2 - 3 = 0$$

(3)

(b) Hence solve, for $-2\pi < x \leqslant 2\pi$

$$4\tan x + 3\cot\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right) = 0$$

giving your answers in terms of π

(4)

Question 3 continued

(Total for Question 3 is 7 marks)

4. Giving your answer using set notation, use algebra to find the complete set of values of x for which

$$|x^2 - 2| > 2x$$

(5)

Question 4 continued

(Total for Question 4 is 5 marks)

5. (i) Given that **a** and **b** are non-zero vectors, find the value of

$$\mathbf{a}.(\mathbf{b} \times \mathbf{a})$$

giving a reason for your answer.

(2)

(ii) Given vectors **a**, **b** and **c** such that $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, where $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{b} \neq \mathbf{c}$, show that

$$\mathbf{b} - \mathbf{c} = \lambda\mathbf{a}$$

where λ is a scalar, giving a reason for your answer.

(2)

Question 5 continued

(Total for Question 5 is 4 marks)

6.

$$\frac{dy}{dx} = x^2 - y^2 \qquad \text{(I)}$$

Given that $y = 1$ at $x = 0$

(a) use the approximation $\left(\frac{dy}{dx}\right)_n \approx \frac{y_{n+1} - y_n}{h}$ with a step length of 0.05 to estimate the value of y at $x = 0.1$

(5)

(b) Use the differential equation (I) to find an expression for $\frac{d^3y}{dx^3}$

(3)

(c) Hence, for the differential equation (I), find the series solution for y in ascending powers of x, up to and including the term in x^3

(5)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 13 marks)

7. A savings account has a fixed annual interest rate of $r\%$ which is compounded n times over the year.

The amount of money in this savings account after one year, £A, is modelled by the equation

$$A = P\left(1 + \frac{r}{100n}\right)^n$$

where £P is the amount in the account at the start of the year.

A student has £1000 in this account at the start of the year and they neither add money to, nor withdraw money from this account.

The annual interest rate is 5%

(a) If the interest is compounded 12 times a year, use the model to show, to the nearest penny, that the amount in the savings account after one year will be £1051.16

(1)

(b) Use L'Hospital's rule to show that $\lim_{n\to\infty}\left(1 + \frac{r}{100n}\right)^n = e^{\frac{r}{100}}$

(6)

(c) Use the model to find the maximum possible amount that the student could have in their savings account after one year.

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

8. A particle is moving in a straight line. The displacement, x m, of the particle relative to a fixed origin O, after it has been moving for t seconds, is modelled by the differential equation

$$t\frac{\mathrm{d}x}{\mathrm{d}t} + 2t^2x = x(2t + 1)$$

(a) Show that the transformation $x = wt$ transforms this equation into the equation

$$\frac{\mathrm{d}w}{\mathrm{d}t} + 2wt - 2w = 0$$

(3)

(b) Hence show that the general equation for the displacement of the particle is

$$x = At\mathrm{e}^{2t-t^2}$$

(5)

When the particle has been moving for 2 seconds, it has a displacement of 10 m.

(c) Find the particular solution for the displacement of the particle.

(1)

(d) Find the maximum displacement of the particle, giving your answer to the nearest centimetre.

(4)

(e) Comment on the displacement of the particle in the long term, as predicted by the model, giving a reason for your answer.

(2)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 15 marks)

9.

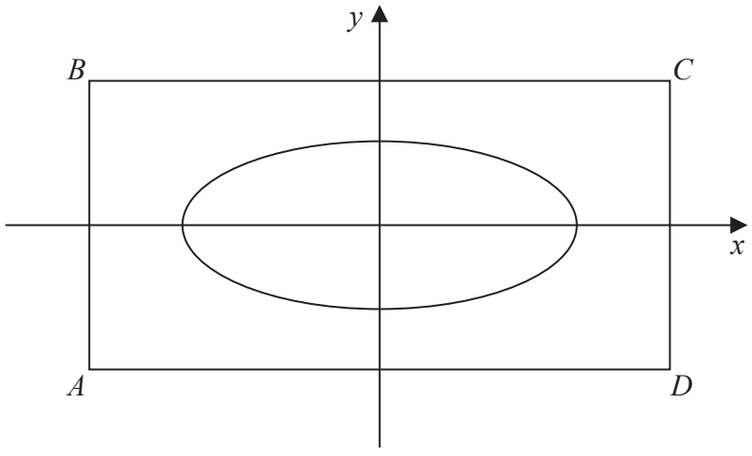

Figure 1

Figure 1 shows the plan for a rectangular garden *ABCD*.
In the middle of the garden is a large pond that may be modelled as an ellipse.
The length of the major axis of the ellipse is twice the length of the minor axis of the ellipse. The line *AB* and the line *CD* are modelled as the directrices of the ellipse.
The ellipse and the rectangle *ABCD* lie in the same horizontal plane.

Given that the length of the garden, *AD*, is $\frac{16}{3}\sqrt{3}$ metres,

(a) find an equation of the ellipse.

(6)

Two water features, modelled as particles, are to be placed in the pond. The sum of the horizontal distances from the water features to any point on the edge of the pond is constant.

(b) Find the coordinates of the points at which the water features are to be placed, according to the model.

(2)

Gnomes, modelled as particles, are to be placed on the edge of the pond. Each gnome will be exactly 2 m from a water feature.

(c) Find all the possible coordinates for the gnomes.

(5)

Question 9 continued

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

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