9FM0/4C: Further Mechanics 02 Mark scheme

Question	Scheme	Marks	AOs
1	Tension in the string: $T = \frac{\lambda x}{a} = 4x$	B1	3.4
	Circular motion: $T = mr\omega^2$	M1	3.4
	$= \frac{1}{2}(x+2)\times 1.5^2$	A1	1.1b
	$\Rightarrow 4x = \frac{1}{2}(2+x)1.5^2$, $\frac{32}{9}x = x+2$	M1	1.1b
	$x = \frac{18}{23}$, $r = 2\frac{18}{23}$ (m)	A1	1.1b
		(5)	
			Total 5 marks

Notes:

1	B1	Correct application of Hooke's law
	1st M1	Resolve towards centre – angular speed form
	1st A1	Correct substituted unsimplified
	2nd M1	Form equation in *x* (or *r*)and solve
	2nd A1	Correct radius. 2.8 or better

Question	Scheme	Marks	AOs
2(a)	<table><tr><td></td><td>*ABCD*</td><td>*EFG*</td><td>*T*</td></tr><tr><td>Mass ratio</td><td>$30a^2$</td><td>$4.5a^2$</td><td>$25.5a^2$</td></tr><tr><td>C of M from *AB*</td><td>$2.5a$</td><td>$2a$</td><td>x</td></tr></table>		
	Mass ratios	B1	1.2
	Distances	B1	1.1b
	Moments equation	M1	2.1
	$30a^2 \times 2.5a - 4.5a^2 \times 2a = 25.5a^2 \times x$	A1	1.1b
	$x = \frac{75-9}{25.5}a\left(=\frac{2\times 66}{51}a\right) = \frac{44}{17}a$ **Given Answer**	A1*	2.2a
		(5)	
2(b)	Moments about *A*: $85 \times \frac{44}{17}a = F \times 6a$	M1	3.1a
	$F = \frac{85 \times 44}{17 \times 6} = \frac{110}{3}$	A1	1.1b
	Use of Pythagoras: $R^2 = 85^2 + \left(\frac{110}{3}\right)^2$	M1	1.1b
	$\lvert R \rvert = 92.6$ (N)	A1	1.1b
		(4)	
		Total 9 marks	

Notes:

2a	1st B1	Mass ratios (all 3)
	2nd B1	Distances from *AB* or from a parallel axis
	M1	Moments about *AB* or a parallel axis. Terms dimensionally consistent. Must be subtracting.
	1st A1	Correct unsimplified equation
	2nd A1*	Show sufficient working to justify given answer
2b	1st M1	Moments about *A*. Dimensionally correct. Condone use of 85g for 85.
	1st A1	Correct *F* – any equivalent form.
	1st M1	Use of Pythagoras with their *F* to find resultant. Condone use of 85g for 85.
	2nd A1	93 or better (92.57129……)

Question	Scheme	Marks	AOs
3(a)	Extension at equilibrium: $2g = \frac{49e}{0.8}$ $\left(e = 0.32(\text{m})\right)$	M1	3.1a
	Equation of motion about equilibrium position:	M1	3.1a
	$2\ddot{x} = 2g - \frac{49(x+e)}{0.8}$ $\left(= -\frac{49x}{0.8}\right)$	A1ft	1.1b
	$\ddot{x} = -\frac{245}{8}x$	A1	1.1b
	which is of the form $\ddot{x} = -\omega^2 x \Rightarrow$ SHM	A1*	3.2a
		(5)	
3(b)	Amplitude $= 1.4 - 0.8 -$ their $0.32 (= 0.28)$	B1ft	2.2a
	Displacement from equilibrium: $x = 0.28\cos\sqrt{\frac{245}{8}}t\,(= 0.08)$	M1	3.4
	Solve for t and double $(t = 0.231...)$	M1	3.1a
	Total time $= 0.46$ (s)	A1	1.1b
		(4)	
			Total 9 marks

Notes:

3a	1st M1	Use the model to find the extension in the spring at equilibrium or an expression for *e*
	2nd M1	Use the model to form the equation for the motion about the equilibrium position. All terms required. Must be dimensionally correct. Condone sign errors
	1st A1ft	Equation with at most one error in *e* or their *e*
	2nd A1	Correct simplified equation
	3rd A1*	CSO: Explain how the result of the working demonstrates the given answer
3b	B1ft	Correct amplitude – follow their *e*
	1st M1	Use the model to write down formula for displacement – follow their a, ω Could be working from the equilibrium position or from *B*.
	2nd M1	Complete method to find total time. Must be working in radians
	A1	Correct answer 0.46, 0.463 only

Question	Scheme	Marks	AOs
4(a)	Total mass $= \int_0^{30} \pi y^2 \times \frac{x}{100}\mathrm{d}x \left(= \frac{\pi}{36}\int_0^{30} \frac{x^3}{100}\mathrm{d}x\right)$	M1	2.1
	$= \frac{\pi}{36}\left[\frac{x^4}{400}\right]_0^{30}$	A1	1.1b
	$= \frac{\pi}{36} \times \frac{810000}{400} = \frac{225\pi}{4}$ (kg) *	A1*	1.1b
		(3)	
(b)	Take moments about the vertex: $\int_0^{30} x \times \pi y^2 \times \frac{x}{100}\mathrm{d}x$	M1	3.4
	$= \frac{\pi}{36}\left[\frac{x^5}{500}\right]_0^{30} \ (= 1350\pi)$	A1ft	1.1b
	$\Rightarrow 1350\pi = \frac{225\pi}{4} d$	M1	3.4
	$d = 24$ (m)	A1	1.1b
		(4)	
	Total 7 marks		

Notes:

4a	M1	Use integration (convincing attempt – at least one power increases)
	1st A1	Correct integration
	2nd A1*	Use limits and show sufficient working to justify **given answer**.
4b	1st M1	Use the model to find the moment about the base (usual rules for integration) Follow their y
	1st A1	Correct integration for their $y = mx$
	2nd M1	Use the model to complete the moments equation. Require $\frac{225\pi}{4}$ and their 1350π used correctly.
	2nd A1	Correct only

Question	Scheme	Marks	AOs
5(a)	$F = \frac{k}{x^2}$	M1	3.4
	Substitute $x = R,\ F = mg \Rightarrow mg = \frac{k}{R^2}$	M1	1.1b
	$k = mgR^2 \Rightarrow F = \frac{mgR^2}{x^2}$ *	A1*	2.1
		(3)	
5(b)	$m\ddot{x} = -\frac{mgR^2}{x^2} \Rightarrow v\frac{dv}{dx} = -\frac{gR^2}{x^2}$	M1	3.4
	$\Rightarrow \int v\,dv = \int -\frac{gR^2}{x^2}dx$	M1	1.1b
	$\frac{1}{2}v^2 = \frac{gR^2}{x}(+C)$	A1	1.1b
	$\frac{1}{2}gR - \frac{1}{2}(U)^2 = \frac{gR^2}{3R} - \frac{gR^2}{R}$	M1	3.1a
	$(U)^2 = gR + \frac{4}{3}gR,\ U = \sqrt{\frac{7gR}{3}}$	A1	1.1b
		(5)	
5(c)	Appropriate refinement	B1	3.5c
		(1)	
			Total 9 marks

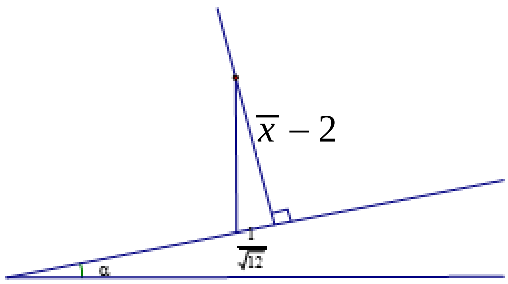
Notes:

5a	1st M1	Use the model to express F in terms of x
	2nd M1	Use $x = R$ to determine the value of k
	A1*	Show sufficient working to justify given answer
5b	1st M1	Use the model to write down the equation of motion for the rocket as a differential equation in v and x
	2nd M1	Separate variables and integrate
	1st A1	Correct integration (do not need to see limits or constant of integration)
	3rd M1	Complete strategy to find U
	2nd A1	Any equivalent form
5c	B1	e.g. do not model the rocket as a particle, take air resistance into account, consider the weight of the fuel in the rocket (which reduces).

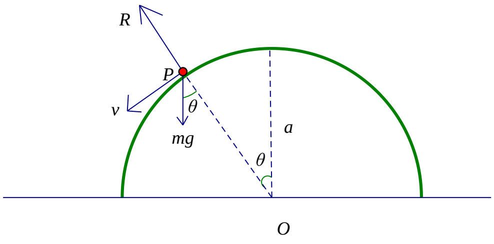
Question	Scheme	Marks	AOs
6(a)	Differentiation: $v\dfrac{dv}{dx}$ or $\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$	M1	2.5
	$=\left(9-\dfrac{3}{x}\right)\times\dfrac{3}{x^2}=\dfrac{27}{x^2}-\dfrac{9}{x^3}$	A1	1.1b
	Substitute for x to find a	M1	1.1b
	$x=3 \Rightarrow a=\dfrac{8}{3}$ (m s^{-2})	A1	1.1b
		(4)	
6(b)	Over all strategy to solve the problem	M1	3.1a
	$v=9-\dfrac{3}{x}=\dfrac{dx}{dt}\left(=\dfrac{9x-3}{x}\right)$	M1	3.4
	$\Rightarrow \int 9dt=\int\dfrac{9x}{9x-3}dx=\int 1+\dfrac{1}{3x-1}dx$	M1	2.1
	$9t=x+\dfrac{1}{3}\ln(3x-1)(+C)$	A1ft A1ft	1.1b 1.1b
	$\Rightarrow 9T=(3-1)+\dfrac{1}{3}\ln\dfrac{9-1}{3-1}$, $T=\dfrac{2}{9}+\dfrac{1}{27}\ln 4$ *	A1*	2.2a
		(6)	
			Total 10 marks

Notes:

6a	1st M1	Complete strategy involving selection of appropriate form for acceleration, differentiation and substitution.
	2nd M1	Differentiate to obtain acceleration
	1st A1	Any equivalent form
	2nd A1	Correct answer. 2.3 or better
6b	M1	Complete strategy e.g. form and solve differential equation and use limits
	M1	Form differential equation in x and t
	M1	Separate variables and integrate. Accept equivalent forms
	A1	At most one error – follow their partial fractions of form $A+\dfrac{B}{3x-1}$
	A1	All correct – follow their partial fractions of form $A+\dfrac{B}{3x-1}$
	A1*	Show sufficient working to deduce **given answer**

Question	Scheme	Marks	AOs
7(a)	$\pi\int\frac{1}{16-(x-4)^2}\mathrm{d}x=\pi\int\frac{1}{x(8-x)}\mathrm{d}x=\frac{\pi}{8}\int\frac{1}{x}+\frac{1}{8-x}\mathrm{d}x$	M1	2.1
	$=\frac{\pi}{8}\ln\frac{x}{8-x}(+C)$	A1	1.1b
	Use of limits to find volume	M1	1.1b
	Volume $=\frac{\pi}{8}\left(\ln\frac{7}{1}-\ln\frac{2}{6}\right)=\frac{\pi}{8}\ln\frac{42}{2}=\frac{\pi}{8}\ln 21$	A1	2.2a
	$\pi\int\frac{x}{16-(x-4)^2}\mathrm{d}x=\pi\int\frac{1}{8-x}\mathrm{d}x$	M1	2.1
	$=-\pi\ln(8-x)(+C)$	A1	1.1b
	$=-\pi\ln\frac{1}{6}=\pi\ln 6$	A1	1.1b
	Correct strategy to find positon of centre of mass	M1	3.1a
	$\bar{x}=\frac{\pi\ln 6}{\frac{\pi}{8}\ln 21}=\frac{8\ln 6}{\ln 21}$ *	A1*	2.2a
		(9)	
7(b)			
	Use of $\frac{1}{\sqrt{12}}$ and $\bar{x}-2$	B1	1.1b
	About to topple so c of m vertically above the tipping point: $\tan\alpha=\frac{1/\sqrt{12}}{\bar{x}-2}$	M1	2.2a
	$\alpha=6.08....$	A1	1.1b
		(3)	
		Total 12 marks	

Notes:		
7a	M1	Use of $\int \pi y^2 \mathrm{d}x$ and correct use of partial fractions to reach a recognised form for integration or correct application of formula. Condone if π not seen
	A1	Any equivalent form. Condone if π not seen
	M1	Use of limits. π must be used.
	A1	Any equivalent form
	M1	Integration of y^2x wrt x. Accept if π not seen. The Q asks for the exact value, so must see exact working.
	A1	Correct integration. Accept with no π and no constant of integration
	A1	Any equivalent form
	A1*	Deduce the **given answer**. Ignore any decimal working after exact answer seen
	M1	Use of $\bar{x} = \frac{\int \pi y^2 x \mathrm{d}x}{\int \pi y^2 \mathrm{d}x}$ with their value for $\int \pi y^2 x \mathrm{d}x$
7b	B1	Correct triangle. 0.2886 and 2.708
	M1	Deduce the position for toppling & use trig to find α
	A1	Accept $\alpha < 6.1°$, $\alpha < 6(.0)°$ or equivalent (0.106 rads)

Question	Scheme	Marks	AOs
8(a)			
	Complete strategy	M1	3.1a
	KE gained = GPE lost	M1	2.1
	$\frac{1}{2}\times mv^2 = mg\left(a - a\cos\theta\right)$	A1	1.1b
	Circular motion: $\frac{mv^2}{a} =$ resultant force towards centre	M1	3.1a
	$\frac{mv^2}{a} = mg\cos\theta - R$	A1	1.1b
	$mg\cos\theta - R = \frac{2}{a}mg\left(a - a\cos\theta\right)$ $\Rightarrow R = 3mg\cos\theta - 2mg = mg\left(3\cos\theta - 2\right)$ *	A1*	2.2a
		(6)	
8(b)	When P leaves the surface, $R = 0$	M1	2.4
	$\Rightarrow \cos\theta = \frac{2}{3}$	A1	2.2a
		(2)	
8(c)	Complete strategy	M1	3.1a
	Conservation of energy top to plane	M1	2.1
	$\frac{1}{2}\times mv^2 = mg\times a \qquad v = \sqrt{2ga}$	A1	1.1b
	Horizontal component $= \cos\theta\times\left(\text{speed on leaving sphere}\right)$	M1	3.1a
	$= \sqrt{\frac{2ga}{3}}\times\frac{2}{3}$	A1	1.1b
	$\Rightarrow \cos\alpha = \frac{\sqrt{\frac{2ga}{3}}\times\frac{2}{3}}{\sqrt{2ga}}\left(=\frac{2}{3\sqrt{3}}\right)$ $\Rightarrow$ downwards at $67.4°$ to the horizontal or downwards at $22.6°$ to the upward vertical	A1	2.2a
		(6)	
			Total 14 marks

Notes:		
8a	M1	Complete strategy: conservation of energy, circular motion and condition for *P* to leave the circle.
	M1	Energy equation. Condone trig confusion. Must be dimensionally correct.
	A1	Correct unsimplified equation
	M1	Equation for circular motion. All terms required. Condone sign errors and sin/cos confusion.
	A1	Correct unsimplified equation
	A1*	Form equation in R, g and θ only .
8b	M1	Any equivalent justification
	A1	Any equivalent form. 0.67 or better
8c	M1	Complete strategy to find two of horizontal component, vertical component and velocity (the sides of the velocity triangle) and use trig to find the direction
	M1	First side of triangle.
	A1	Correct unsimplified value
	M1	Second side of triangle
	A1	Correct unsimplified value
	A1	Deduce correct final answer. Answer needs to make direction clear either in words or in a diagram