9FM0/3C: Further Mechanics 01 Mark scheme

Question	Scheme	Marks	AOs
1(a)	Driving force $= \dfrac{12000}{V}$ (N)	B1	3.1b
	No resultant force $\Rightarrow 200 + V^2 = \dfrac{12000}{V} \Rightarrow V^3 + 200V - 12000 = 0$	M1	3.4
	$(V-20)(V^2+20V+600)=0 \Rightarrow V=20$ is a solution*	A1*	2.2a
	$b^2 - 4ac = 400 - 2400 < 0$ so no more real roots, $V = 20$ is the only solution.	A1	2.4
		(4)	
(b)	Equation of motion:	M1	3.4
	$\dfrac{15000}{v} - 200 - v^2 - 750g\sin\theta = 750a$	A1 A1	1.1b 1.1b
	$\Rightarrow 1200 - 50g = 750a$	M1	1.1b
	$a = 0.95$ m s^{-2} (0.947)	A1	1.1b
		(5)	
			(9 marks)

Notes:

1a	B1	Use $P = Fv$ to find the driving force
	M1	Use the model to form an equation in V
	A1*	Solve equation to obtain solution $V = 20$ (complex roots $-10 \pm 10\sqrt{5}\text{i}$)
	A1	CSO. Justification that $V = 20$ is the only real solution e.g. by considering determinant of quadratic factor, completing the square or stating all 3 roots and confirming that only one root is real
SR	A candidate who verifies that $V = 20$ is a solution can score 2/4:	
	B1	Use $P = Fv$ to find the driving force
	M1	Complete method to show that there is no resultant force when that $V = 20$
1b	M1	Use the model to form the equation of motion of the van. All terms required. Condone sign errors and sin/cos confusion
	A1	Unsimplified equation with at most one error
	A1	Correct unsimplified equation
	M1	Substitute for v and trig and solve for a
	A1	Accept 2s.f. or 3s.f. (9.8 used)

Question	Scheme	Marks	AOs
2	EPE $=\dfrac{\frac{3}{4}mgl^2}{2l}$ or EPE $=\dfrac{\frac{3}{4}mg\left(\frac{9l}{25}\right)^2}{2l}$	B1	3.4
	Gain in GPE $= mg\times\frac{2}{5}l\sin\alpha\left(=\frac{2}{13}mgl\right)$	B1	1.1b
	Work done against friction $=\mu mg\cos\alpha\times\frac{2l}{5}\left(=\mu\times\frac{24}{65}mgl\right)$	B1	1.1b
	Work-Energy equation	M1	3.1a
	$\frac{3mgl}{8}\left(1-\frac{9}{25}\right)=\frac{2}{5}mgl\sin\alpha+\frac{2}{5}\mu mgl\cos\alpha$ $\left(\frac{6mgl}{25}=\frac{2}{13}mgl+\mu\times\frac{24}{65}mgl\right)$	A1	1.1b
	Substitute trig and solve for μ: $\left(\frac{6}{25}-\frac{2}{13}=\frac{24}{65}\mu\right)$		
	$\mu=\frac{7}{30}$ (0.233)	A1	1.1b
		[6]	
			(6 marks)

Notes:

2	B1	Correct unsimplified expression for EPE at *B* or at *C*
	B1	Correct unsimplified expression for GPE gained *B* to *C*
	B1	Correct unsimplified expression for WD against friction *B* to *C*
	M1	All terms required. Condone sign errors and sin/cos confusion.
	A1	Correct unsimplified equation
	A1	0.23 or better (*g* cancels)

Question	Scheme	Marks	AOs
3(a)	Impulse momentum equation	M1	2.1
	$\mathbf{I} = 3(-\mathbf{i}+\lambda\mathbf{j}) - 3(2\mathbf{i}+\mathbf{j})\left(= -9\mathbf{i} + (3\lambda - 3)\mathbf{j}\right)$	A1	1.1b
	Magnitude of the impulse	M1	1.1b
	$130 = 81 + (3\lambda - 3)^2$ Follow their **I**	A1ft	1.1b
	$3\lambda - 3 = (\pm)7$, $\lambda = \frac{10}{3}$	M1	2.2a
	$\mathbf{I} = -9\mathbf{i} + 7\mathbf{j}$ (Ns)	A1	1.1b
		(6)	
3(b)	Use of scalar product: $(2\mathbf{i}+\mathbf{j})\cdot\left(-\mathbf{i}+\frac{10}{3}\mathbf{j}\right) = \frac{4}{3}$	M1	3.1a
	$\cos\theta = \dfrac{\frac{4}{3}}{\sqrt{\frac{109}{9}}\sqrt{5}} \left(= \frac{4}{\sqrt{545}}\right)$ follow their λ	A1ft	1.1b
	$\theta = 80.1$	A1	1.1b
		(3)	
3(b) alt	Use trig to find 2 relevant angles: $\tan^{-1}\pm\frac{10}{3}$, $\tan^{-1}\frac{1}{2}$	(M1)	3.1a
	$73.30°$ or $106.70°$, $26.57°$	(A1)	1.1b
	$\theta = 80.1$	(A1)	1.1b
		(3)	
			(9 marks)

Notes:

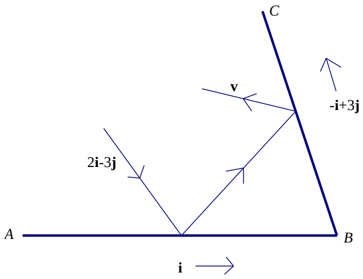
3a	M1	Use impulse momentum to find the impulse in terms of λ
	A1	Correct unsimplified equation
	M1	Use Pythagoras and the given modulus
	A1ft	Correct unsimplified expression using their **I**
	M1	Solve for λ (or $3\lambda - 3$) and find **I**
	A1	Correct answer
3b	M1	Complete strategy, using vectors or equivalent, to find relevant angle Could be working with velocity or momentum.
	A1ft	Single trig ratio or all relevant angles. Follow their λ
	A1	80 or better

Question	Scheme	Marks	AOs
4(a)	Complete strategy to find k	M1	3.1a
	Resolve vertically: $2T\cos\theta = mg \;\; 2T\times\frac{3}{5} = mg$	B1	1.1b
	Hooke's Law and equiibrium: $T = kmg\frac{l}{4l} \Rightarrow \;\; 2\frac{kmg}{4}\times\frac{3}{5} = mg$	M1	2.1
	$\Rightarrow k = \frac{10}{3}$ *	A1*	2.2a
		(4)	
4(b)	Equation of motion:	M1	3.1a
	$2T\cos\alpha - mg = ma$, $\quad 2\times\frac{\frac{10}{3}mg\times\frac{8}{3}l}{4l}\times\frac{4}{5} - mg = ma$	A1	1.1b
	$\left(\left(\frac{32}{9}-1\right)mg = ma\right)$		
	$a = \frac{23}{9}g$	A1	1.1b
		(3)	
4(c)	Conservation of energy:	M1	3.1a
	$2\times\frac{\frac{10}{3}mg\times\frac{64}{9}l^2}{2\times 4l} = mg\times\frac{16}{3}l + \frac{1}{2}mv^2$	A1	1.1b
	$v = \sqrt{\frac{32gl}{27}}\left(= \frac{4}{3}\sqrt{\frac{2gl}{3}}\right)$	A1	1.1b
		(3)	
4(d)	Any sensible reason in context	B1	3.5b
		(1)	
			(11 marks)

Notes:

4a	M1	Complete strategy e.g. resolve vertically to find T and use Hooke's law
	B1	Correct substituted equation in T
	M1	Correct use of Hooke's law and equilibrium to find the tension in the string
	A1*	Draw the information together to deduce the **given result**

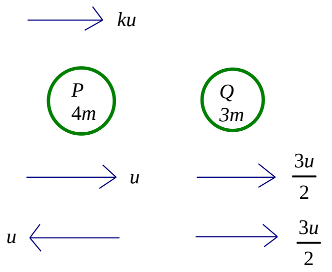
4b	M1	Use the model to form the equation of motion of *P*. Need all terms. Dimensionally correct. Condone sign errors and sin/cos confusion.
	A1	Correct substituted unsimplified.
	A1	25 or 25.0 m s^{-2} if 9.8 used.
4c	M1	Use the model to write down the equation for conservation of energy: EPE lost = GPE gained + KE gained
	A1	Any unsimplified equivalent
	A1	Accept any equivalent simplified form or $3.4\sqrt{l}$
4d	B1	e.g. The pebble has dimensions, so the instant of crossing *AB* is not well-defined Some of the string could be taken up attaching the pebble Accuracy of the measurement of the speed

Question	Scheme	Marks	AOs
5			
	After first impact: parallel to *AB* $2\mathbf{i}$	B1	2.1
	Use of impact law perpendicular to *AB*	M1	3.4
	$-\frac{1}{2}(-3\mathbf{j}) = \frac{3}{2}\mathbf{j}$	A1	1.1b
	Strategy to find final velocity	M1	3.1b
	Second impact: parallel to *BC* $\mathbf{v}\cdot\left(\frac{-\mathbf{i}+3\mathbf{j}}{(\sqrt{10})}\right)$	M1	3.1b
	$\left(\left(2\mathbf{i}+\frac{3}{2}\mathbf{j}\right)\cdot\left(\frac{-\mathbf{i}+3\mathbf{j}}{(\sqrt{10})}\right)=\frac{5}{2\sqrt{10}}\right)$ follow their **v**	A1ft	1.1b
	Component of velocity $=\frac{5}{2\sqrt{10}}\times\left(\frac{-\mathbf{i}+3\mathbf{j}}{\sqrt{10}}\right)=\frac{1}{4}(-\mathbf{i}+3\mathbf{j})$	A1	1.1b
	Vector perpendicular to the wall $(3\mathbf{i}+\mathbf{j})$	B1	3.1b
	Use of impact law:	M1	3.4
	$-\frac{1}{3}\left(2\mathbf{i}+\frac{3}{2}\mathbf{j}\right)\cdot\frac{(3\mathbf{i}+\mathbf{j})}{(\sqrt{10})}$ Follow their velocity and their perpendicular vector	A1ft	1.1b
	Component of velocity $=-\frac{5}{2\sqrt{10}}\times\left(\frac{3\mathbf{i}+\mathbf{j}}{\sqrt{10}}\right)=-\frac{1}{4}(3\mathbf{i}+\mathbf{j})$	A1	1.1b
	$\Rightarrow \mathbf{v}=\frac{1}{4}(-\mathbf{i}+3\mathbf{j})-\frac{1}{4}(3\mathbf{i}+\mathbf{j})$ (sum of their components)		
	$=\left(-\mathbf{i}+\frac{1}{2}\mathbf{j}\right)$ (m s^{-1}) *	A1*	2.2a
		(12)	
5 alt	***For the last 9 marks***		
	Strategy to find final velocity	M1	
	Perpendicular to $-\mathbf{i}+3\mathbf{j}$ is $-3\mathbf{i}-\mathbf{j}$	B1	
	Find components of the initial velocity parallel and perpendicular to $-\mathbf{i}+3\mathbf{j}$: $\mathbf{v}=p(-\mathbf{i}+3\mathbf{j})+q(-3\mathbf{i}-\mathbf{j})$	M1	

	$\begin{cases} 2 = -p - 3q \\ \frac{3}{2} = 3p - q \end{cases} \Rightarrow p = \frac{1}{4}$	A1	
	$q = -\frac{3}{4}$, $\left(\mathbf{v} = \frac{1}{4}(-\mathbf{i} + 3\mathbf{j}) - \frac{3}{4}(-3\mathbf{i} - \mathbf{j})\right)$	A1	
	Impact law perpendicular to plane: $\pm\frac{1}{3} \times -\frac{3}{4}(-3\mathbf{i} - \mathbf{j})$	M1	
	Follow their perpendicular component	A1ft	
	Parallel component: $\frac{1}{4}(-\mathbf{i} + 3\mathbf{j})$ Follow their parallel component	A1ft	
	Final velocity $= \frac{1}{4}(-\mathbf{i} + 3\mathbf{j}) + \frac{1}{4}(-3\mathbf{i} - \mathbf{j}) = -\mathbf{i} + \frac{1}{2}\mathbf{j}$ *	A1*	
			(12 marks)

Notes:

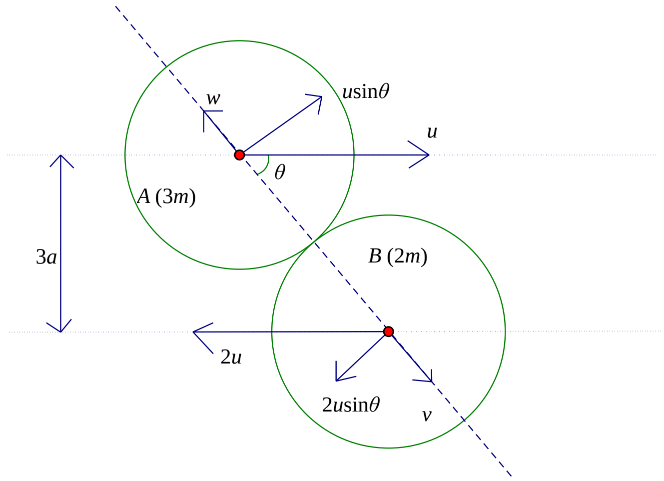
5	B1	Conservation of component parallel to the first wall
	M1	Use the impact law on the model to find the component of the velocity perpendicular to *AB* after the impact
	A1	Correct value
	M1	Complete strategy to find final velocity: find components parallel and perpendicular to *BC* and add.
	M1	Scalar product of their velocity with a vector parallel to *BC*. Condone missing modulus.
	A1	Correct unsimplified (follow their $2\mathbf{i} + \frac{3}{2}\mathbf{j}$)
	A1	Correct parallel component
	B1	Any parallel vector
	M1	Correct use of the model and the impact law to find the magnitude of the perpendicular component. Condone missing modulus.
	A1ft	Correct unsimplified. Follow their $2\mathbf{i} + \frac{3}{2}\mathbf{j}$ and their perpendicular vector
	A1	Correct perpendicular component
	A1*	Combine the components to deduce the **given answer**

Question	Scheme	Marks	AOs
6(a)			
	Two correct possibilities identified	B1	2.1
	Form and solve a pair of simultaneous equations in k and e	M1	3.1a
	Use of CLM:	M1	3.1a
	$4mu + 3m \times \dfrac{3u}{2} = 4mku$ or $-4mu + 3m \times \dfrac{3u}{2} = 4mku$	A1	1.1b
	Use of impact law:	M1	3.1a
	$\dfrac{3}{2}u - u = e \times ku$ or $\dfrac{3}{2}u + u = e \times ku$	A1	1.1b
	$\dfrac{17}{2} = 4k$ and $\dfrac{1}{2} = ek \Rightarrow k = \dfrac{17}{8}$, $e = \dfrac{4}{17}$	A1	1.1b
	Second pair of simultaneous equations	M1	3.4
	Both equations correct	A1	1.1b
	$\dfrac{1}{2} = 4k$ and $\dfrac{5}{2} = ek \Rightarrow k = \dfrac{1}{8}$		
	$e = 20$ impossible since max $e = 1$	M1	1.1b
	Convincing argument to support just one possible value for k*****.	A1*	2.2a
	Alternative for last 4 marks:		
	Second CLM equation	M1	3.4
	$\dfrac{1}{2} = 4k \Rightarrow k = \dfrac{1}{8}$	A1	1.1b
	$k = \dfrac{1}{8} \Rightarrow$ both particles gain KE, which is impossible	M1	1.1b
	Convincing argument to support just one possible value for k*****.	A1*	2.2a
		(11)	

6(b)	KE lost = difference of two KEs	M1	3.1a
	$= \frac{1}{2}\times 4m\times(ku)^2 - \frac{1}{2}\times 4m\times u^2 - \frac{1}{2}\times 3m\times\left(\frac{3}{2}u\right)^2$ $= mu^2\left(2k^2 - 2 - \frac{27}{8}\right)$	A1ft	1.1b
	$= \frac{117}{32}mu^2$ or equivalent	A1	1.1b
		(3)	
			(14 marks)

Notes:

6a	B1	Identify all possible options from given information
	M1	Complete strategy to find a pair of values for k and e
	M1	Correct use of CLM. All terms needed. Condone sign errors. Dimensionally correct
	A1	Correct unsimplified equation (for either option)
	M1	Correct use of impact law.
	A1	Correct unsimplified equation (for the same option)
	A1	Correct solution for one pair of k and e
	M1	Form second pair of simultaneous equations to fit the model.
	A1	Both equations correct unsimplified
	M1	Correct reasoning for elimination of one pair of values
	A1*	CSO. Deduce the **given result** having considered all the options.
6b	M1	Complete strategy to find an expression in m, (k) and u for the KE lost.
	A1ft	Correct unsimplified expression in k or their k
	A1	$3.7mu^2$ or better

Question	Scheme	Marks	AOs
7(a)			
	Complete strategy to find impulse	M1	3.1a
	CLM parallel to line of centres	M1	3.1a
	$3mw - 2mv = 2m.2u\cos\theta - 3m.u\cos\theta$ $\left(= mu\cos\theta\right)$	A1	1.1b
	Use of impact law parallel to line of centres	M1	3.1a
	$w + v = \frac{1}{3}\left(u\cos\theta + 2u\cos\theta\right)\left(= u\cos\theta\right)$	A1	1.1b
	Solve for v or w: $\begin{cases} 3w - 2v = u\cos\theta \\ 2w + 2v = 2u\cos\theta \end{cases} \Rightarrow w = \frac{3}{5}u\cos\theta \quad \left(v = \frac{2}{5}u\cos\theta\right)$	A1	1.1b
	Correct trig ratio used $\left(\cos\theta = \frac{\sqrt{7}}{4}, \quad \sin\theta = \frac{3}{4}\right)$	B1	1.1b
	Magnitude of impulse $= \left\|3m\left(w \pm u\cos\theta\right)\right\|$	M1	3.1a
	$= \left\|3m\left(\frac{3}{5}u\cos\theta + u\cos\theta\right)\right\|$	A1	1.1b
	$= \left\|3m\left(\frac{8}{5}u \times \frac{\sqrt{7}}{4}\right)\right\| = \frac{6\sqrt{7}}{5}mu$ *	A1*	2.2a
		(10)	
7(b)	Component of velocity perpendicular to line of centres $= u\sin\theta$	B1	3.4
	Speed $= \sqrt{\left(u\sin\theta\right)^2 + \left(\frac{3}{5}u\cos\theta\right)^2}$ for their w	M1	2.1
	$= u\sqrt{\frac{9}{16} + \frac{9 \times 7}{25 \times 16}} = \frac{3\sqrt{2}}{5}u$	A1	1.1b
		(3)	

7(c)	Impulse only acts along the line of centres	B1	3.5b
		(1)	
			(14 marks)

Notes:

7a	M1	Over all strategy: form and solve simultaneous equations and use impulse/momentum.
	M1	Use of CLM parallel to l of c. All terms needed. Condone sign errors and sin/cos confusion.
	A1	Correct unsimplified equation
	M1	Must be used the right way round. Follow their components of u and $2u$.
	A1	Correct unsimplified equation
	A1	v or w correct in terms of u and θ
	B1	Correct trig ratio seen or implied
	M1	Magnitude of impulse on either particle. Must be using change in component of velocity.
	A1	Correct unsimplified in terms of m, u and θ
	A1*	Substitute trig values and deduce the **given result**
7b	B1	Use conservation of component of velocity perpendicular to line of centres
	M1	Use of Pythagoras to combine the components parallel and perpendicular to the line of centres. Follow their w.
	A1	Any equivalent simplified form
7c	B1	Any valid modelling assumption – no spin, no friction, no change perpendicular to the line of centres