9FM0/4D: Decision Mathematics 02 Mark scheme

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>1 (a)</td><td>
<table>
<tr><td></td><td></td><td>23</td><td>18</td><td>22</td><td>24</td></tr>
<tr><td></td><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>0</td><td>A</td><td></td><td></td><td></td><td></td></tr>
<tr><td>11</td><td>B</td><td></td><td></td><td>–1</td><td>–7</td></tr>
<tr><td>9</td><td>C</td><td>5</td><td></td><td></td><td></td></tr>
<tr><td>11</td><td>D</td><td></td><td></td><td>–4</td><td></td></tr>
</table>
</td><td>B1
B1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>
<table>
<tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>A</td><td></td><td></td><td></td><td></td></tr>
<tr><td>B</td><td></td><td>$14-\theta$</td><td></td><td>θ</td></tr>
<tr><td>C</td><td></td><td>$6+\theta$</td><td></td><td>$3-\theta$</td></tr>
<tr><td>D</td><td></td><td></td><td></td><td></td></tr>
</table>
<table>
<tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>A</td><td>15</td><td></td><td></td><td></td></tr>
<tr><td>B</td><td>3</td><td>11</td><td></td><td>3</td></tr>
<tr><td>C</td><td></td><td>9</td><td>14</td><td></td></tr>
<tr><td>D</td><td></td><td></td><td></td><td>16</td></tr>
</table>
Entering cell is B4 and exiting cell is C4
</td><td>M1
A1
A1</td><td>2.1
1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(c)</td><td>
<table>
<tr><td></td><td></td><td>23</td><td>18</td><td>22</td><td>17</td></tr>
<tr><td></td><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>0</td><td>A</td><td>X</td><td>14</td><td>4</td><td>13</td></tr>
<tr><td>11</td><td>B</td><td>X</td><td>X</td><td>–1</td><td>X</td></tr>
<tr><td>9</td><td>C</td><td>5</td><td>X</td><td>X</td><td>7</td></tr>
<tr><td>18</td><td>D</td><td>–7</td><td>–8</td><td>–11</td><td>X</td></tr>
</table>
Negative IIs so solution is not optimal
</td><td>M1
A1
A1</td><td>2.1
1.1b
2.4</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>

(d)	(i) The algorithm needs to be adapted because if the stock is increased then total supply > total demand and the problem is then unbalanced (it won't be possible to match the supply to demand)	B1	3.5b
	(ii) For an unbalanced problem a dummy demand point needs to be added… …such that the total supply = total demand with transportation costs of zero	B1 B1	3.5c 3.5c
		(3)	
			(11 marks)

Notes:

(a) B1: correct answer for shadow costs

B1: correct answer for remaining improvement indices

SC for (a) if B0B0 then award B1 for seven or more correct values

(b) M1: a valid route, their most negative II chosen, only one empty square used, θ's balance

A1: cao

A1: correct answer for both the entering cell (B4) and exiting cell (C4)

(c) M1: finding all 8 shadow costs **and** all 9 negative improvement indices **or** sufficient number of shadow costs for at least 1 negative II found– this mark is dependent on the previous M mark in (b) which will therefore indicate a correct mathematical argument leading from the initial solution to the confirmation or not of the optimality of the current solution

A1: cao any negative II from correct working

A1: cso including the correct reasoning that the solution is not optimal because there are negative IIs

(d) B1: cao that problem is unbalanced with understanding of what 'unbalanced' means in this context (e.g. total supply > total demand)

B1: cao that a dummy demand point needs to be added

B1: total supply = total demand **and** zero transportation costs

Question	Scheme	Marks	AOs
2(a)	$\sum x_{\text{A}j}=1$ $\sum x_{\text{B}j}=1$ $\sum x_{\text{C}j}=1$ $\sum x_{\text{D}j}=1$ $\sum x_{i1}=1$ $\sum x_{i2}=1$ $\sum x_{i3}=1$ $\sum x_{i4}=1$	B1 B1	3.3 1.1b
		(2)	
(b)	Minimise $C=29x_{\text{A1}}+15x_{\text{A2}}+32x_{\text{A3}}+27x_{\text{A4}}$ $+33x_{\text{B1}}+25x_{\text{B2}}+30x_{\text{B3}}+31x_{\text{B4}}$ $+40x_{\text{C1}}+43x_{\text{C2}}+37x_{\text{C3}}+34x_{\text{C4}}$ $+30x_{\text{D1}}+20x_{\text{D2}}+27x_{\text{D3}}+37x_{\text{D4}}$	B1	3.3
		(1)	
(c)	Reduce rows $\begin{bmatrix}14&0&17&12\\8&0&5&6\\6&9&3&0\\10&0&7&17\end{bmatrix}$ and then columns $\begin{bmatrix}8&0&14&12\\2&0&2&6\\0&9&0&0\\4&0&4&17\end{bmatrix}$	M1 A1	2.1 1.1b
	$\begin{bmatrix}6&0&12&10\\0&0&0&4\\0&11&0&0\\2&0&2&15\end{bmatrix}$ followed by $\begin{bmatrix}4&0&10&8\\0&2&0&4\\0&13&0&0\\0&0&0&13\end{bmatrix}$	M1 M1 A1	2.1 1.1b 1.1b
	Either A – 2, B – 1, C – 4, D – 3 or A – 2, B – 3, C – 4, D – 1	A1ft	2.2a
		(6)	
		(9 marks)	

Notes:

(a) B1: any four correct (accept equivalent e.g. $x_{\text{A1}}+x_{\text{A2}}+x_{\text{A3}}+x_{\text{A4}}=1$) equations not inequalities

B1: all eight correct

(b) B1: cao

(c) M1: simplifying the initial matrix by reducing rows and then columns

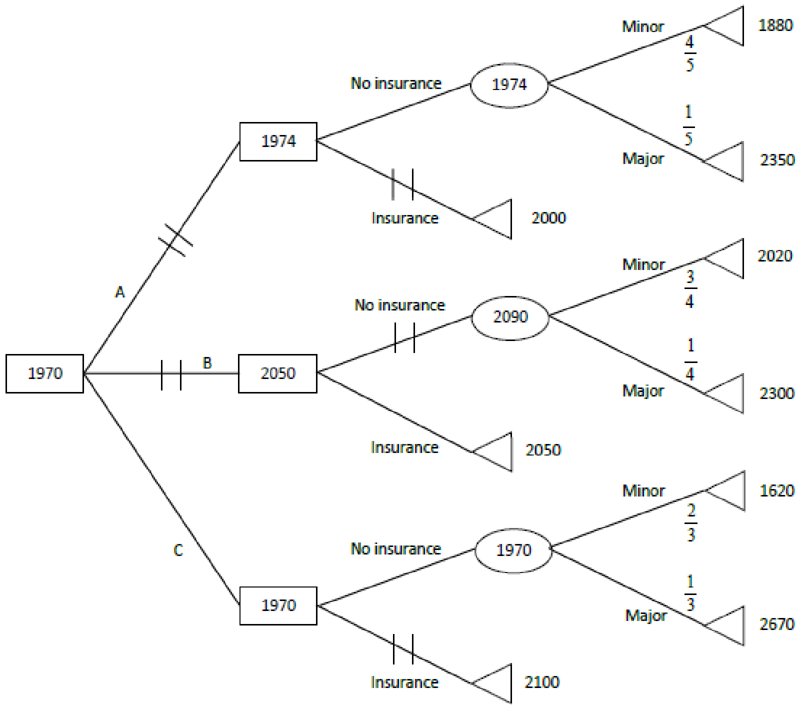
A1: cao

M1: develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 2 lines needed to 3 lines needed

M1: develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed (so getting to the optimal table)

A1: cso on final table (so must have scored all previous marks in this part)

A1ft: correct allocation ft their optimal table (all previous M marks must have been awarded)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>3(a)</td><td>
</td><td>M1
A1
M1
A1
M1
A1</td><td>3.3
1.1b
3.4
1.1b
3.4
1.1b</td></tr>
<tr><td></td><td></td><td>(6)</td><td></td></tr>
<tr><td>(b)</td><td>EMV is £1970</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td>Susie should choose boat C with no insurance</td><td>B1</td><td>3.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td colspan="4" align="right">(8 marks)</td></tr>
</table>

Notes:

(a) M1: tree diagram with at least six end pay-offs, two decision nodes and two chance nodes

A1: correct structure of tree diagram with each arc labelled correctly (including probabilities)

M1: all three chance nodes attempted with one correct

A1: cao for all three chance nodes

M1: at least three decision nodes complete and consistent with their chance nodes

A1: cao for decision nodes including double lines through inferior options

(b) B1: correct EMV

B1: correct analysis in context

Question	Scheme	Marks	AOs
4(a)	maximin	B1	1.2
		(1)	
(b)	(i)		

Stage	State	Action	Destination	Value
4	J	JT	T	20*
	K	KT	T	19*
	L	LT	T	21*
3	F	FJ	J	min (20, 20) = 20*
		FK	K	min (18, 19) = 18
	G	GJ	J	min (20, 20) = 20
		GL	L	min (22, 21) = 21*
	H	HK	K	min (21, 19) = 19*
		HL	L	min (19, 21) = 19*
	I	IK	K	min (21, 19) = 19
		IL	L	min (23, 21) = 21*
2	C	CF	F	min (16, 20) = 16
		CG	G	min (23, 21) = 21*
		CH	H	min (19, 19) = 19
	D	DF	F	min (15, 20) = 15
		DH	H	min (11, 19) = 11
		DI	I	min (16, 21) = 16*
	E	EH	H	min (17, 19) = 17
		EI	I	min (18, 21) = 18*
1	A	AC	C	min (17, 21) = 17*
		AD	D	min (14, 16) = 14
	B	BC	C	min (18, 21) = 18*
		BD	D	min (15, 16) = 15
		BE	E	min (19, 18) = 18*
0	S	SA	A	min (22, 17) = 17
		SB	B	min (24, 18) = 18*

Question	Scheme	Marks	AOs
	(b)(i) table	B1 M1 A1 A1 M1 A1ft M1 A1ft A1	3.1b 3.1b 1.1b 1.1b 1.1b 1.1b 1.1b 1.1b 1.1b
	Maximum weight = 18 tonnes	A1	2.2a
	(ii)Route 1: S – B – E – I – L – T Route 2: S – B – C – G – L – T	B1 B1	1.1b 1.1b
		(12)	
(c)	(i) New maximum weight is 17 tonnes (ii) Route: S – A – C – G – L – T	B1ft B1ft	2.2a 2.2a
		(2)	
			(15 marks)

Notes:
(a) B1: cao **Throughout (b):** **• Condone lack of destination column and/or reversed stage numbers throughout** **• Only penalise incorrect result in value – ie ignore working values** **• Penalise absence of state or action column with first two A marks earned only** **• Penalise empty/errors in stage column with first A mark earned only** **M marks - must bring earlier optimal results into calculations at least once** **Penalise lack of * only once** **B1:** stage 4 correct **M1:** stage 3 completed with 4 states and at least 8 rows. Bod if something in each cell **A1:** any two states in Stage 3 correct **A1:** cao all 4 states correct in Stage 3 (no extra rows) **M1:** stage 2 completed with 3 states and at least 8 rows. Bod if something in each cell **A1ft:** cao all 3 states correct in Stage 2 (no extra rows) on the follow through **M1:** stage 1 completed with 2 states and at least 5 rows. Bod if something in each cell **A1ft:** cao for Stage 1 following through their * values (no extra rows) **A1:** cao for Stage 0 (no follow through for this mark) **A1:** cso answer in context – all previous marks must have been awarded **B1:** any correct route **B1:** both correct routes **(c)** **B1ft:** follow through their value for action SA **B1ft:** follow through their route working backwards from action SA

Question	Scheme	Marks	AOs
5(a)	2	B1	1.1b
		(1)	
(b)	Strategy Q <u>dominates</u> Strategy S Because 2 > 0 and 1 > -1	B1 B1	2.5 2.4
		(2)	
(c)	Row minima: $-3, 1, -2$ max is 1 Column maxima: 3, 3 min is 3 Row maximin (1) $\neq$ Column minimax (3) so not stable	M1 A1 A1	1.1b 1.1b 2.4
		(3)	
(d)	(i) Augment by 3 to make all values non-negative i.e. $\begin{pmatrix} 0 & 6 \\ 5 & 4 \\ 6 & 1 \end{pmatrix}$ For this augmented matrix if player B plays strategy X player A expects $5p_2 + 6p_3$ and if player B plays strategy Y player A expects $6p_1 + 4p_2 + p_3$ (ii) V is less than or equal to each of these two expressions since we need to find the maximum value of the worst possible augmented expected pay-off for each value of p	B1 B1 B1	1.1b 2.4 2.3
		(3)	
(e)	We use an inequality because this is needed to enable the Simplex algorithm to pivot on a row that will increase the value of P	B1	1.2
		(1)	
(f)	(i) $p_2 = \frac{6}{7}$ (ii) Substitute p values to obtain $V \leq \frac{30}{7}$ Value of the game to player B $= -\left(\frac{30}{7} - 3\right) = -\frac{9}{7}$	B1 M1 A1	1.1b 3.4 2.2a
		(3)	
			(13 marks)

Notes:
(a) B1: cao **(b) B1:** correct statement – must include the word ‘dominate’ **B1:** correct inequalities – must be clear that both inequalities must hold **(c) M1:** attempt at row minima and column maxima – condone one error **A1:** correct max(row min) and min(col max) **A1:** correct reasoning that the game is not stable (accept $1 \neq 3$ + statement) **(d) B1:** augment by 3 or new matrix written out **B1:** correct explanation – note that expressions are given in the question **B1:** an understanding that for each value of *p* we are seeking the minimum possible output **(e) B1:** as a minimum accept an answer that implies that an inequality is required so that we can apply the Simplex algorithm **(f) B1:** cao **M1:** substitute their *p* values into both expressions for the upper bound of *V* **A1:** cao for the value of the game to player B

Question	Scheme	Marks	AOs
6(a)	$y_{n+1} = 0.16x_n - 4 \Rightarrow y_n = 0.16x_{n-1} - 4$ therefore $x_{n+1} = 4(0.16x_{n-1} - 4) - 2$	M1	2.1
	$x_{n+1} - 0.64x_{n-1} = -18$ so $k = -18$	A1	2.2a
		(2)	
(b)	aux equation $m^2 - 0.64 = 0 \Rightarrow m = \pm 0.8$	B1	2.1
	$x_n = A(0.8)^n + B(-0.8)^n$	B1	1.1b
	particular solution try $x_n = \lambda$ $\therefore \lambda - 0.64\lambda = -18 \Rightarrow \lambda = \ldots$	M1	1.1b
	$x_n = A(0.8)^n + B(-0.8)^n - 50$	A1ft	1.1b
	$x_1 = -50.8 \Rightarrow A - B = -1$	M1	1.1b
	$y_1 = -11.52 \Rightarrow x_2 = -48.08$	B1	3.1a
	$A(0.8)^2 + B(-0.8)^2 - 50 = -48.08 \Rightarrow A + B = 3$	M1	1.1b
	$A = 1, B = 2 \Rightarrow x_n = (0.8)^n + 2(-0.8)^n - 50$	A1	2.2a
		(8)	
(c)	$y_n = \frac{1}{4}(x_{n+1} + 2) \Rightarrow y_n = \ldots$	M1	3.1a
	$y_n = \frac{1}{4}\left((0.8)^{n+1} + 2(-0.8)^{n+1} - 48\right)$	A1ft	1.1b
		(2)	
(d)	$x_n \to -50$ and $y_n \to -12$	B1ft	2.2a
		(1)	
			(13 marks)

Notes:
(a) M1: attempt to re-write y_n in terms of x_{n-1} and substitute into x_{n+1} **A1:** cao – sufficient working must be shown as lhs is given in the question – allow implicit stating of the value for *k* **(b) B1:** cao for auxiliary equation and corresponding solutions (this mark can be implied by correct complementary function) **B1:** cao **M1:** substitute $x_n = \lambda$ into their second-order recurrence relation and solve for λ **A1ft:** correct general solution following through candidate's value for *k* **M1:** uses $x_1 = -50.8$ to form an equation in *A* and *B* **B1:** cao for x_2 (or equivalent to allow forming of second equation in *A* and *B*) **M1:** setting up a second equation in *A* and *B* **A1:** cao **(c) M1:** uses $x_{n+1} = 4y_n - 8$ and their expression for x_n to find an expression for y_n **A1ft:** cao following through their x_n (dependent on all B and M marks in (b)) **(d) B1ft:** cao following through their x_n and y_n

Question	Scheme	Marks	AOs
7(a)	$8 \leq$ flow in BC ≤ 10	B1 B1	3.4 1.1b
	• Minimum flow into B is 14 (6 from SB and 8 from AB) • Maximum flow along BD is 6 so min flow in BC is 14 – 6 = 8 • Maximum flow out of C is 15 (7 from CT and 8 from CD) • Minimum flow along AC is 5 so max flow in BC is 15 – 5 = 10	M1 A1	2.1 2.4
		(4)	
(b)	$x \leq 10, y \geq 8$	B1ft	2.2a
	$y \geq x$	B1	1.2
		(2)	
		(6 marks)	

Notes:

(a) B1: using the model to obtain one correct value (8 or 10)

B1: cao – with correct inequalities

M1: considering min/max flow at B or C (give bod)

A1: correct explanation regarding min/max flow at B and C

(b) B1ft: correct deduction based on their answer to part (a)

B1: cao **(**$y \geq x$)