9FM0/3D: Decision Mathematics 01 Mark scheme

Question	Scheme	Marks	AOs
1(a)	6 1 9 14 18 $\boxed{7}$ 10 4 17 13 9 14 18 $\boxed{10}$ 17 13 $\underline{7}$ 6 $\boxed{1}$ 4	M1	1.1b
	14 18 $\boxed{17}$ 13 $\underline{10}$ 9 $\underline{7}$ 6 $\boxed{4}$ $\underline{1}$ 18 $\underline{17}$ 14 $\boxed{13}$ $\underline{10}$ 9 $\underline{7}$ 6 $\underline{4}$ $\underline{1}$	A1	1.1b
	18 $\underline{17}$ 14 $\underline{13}$ $\underline{10}$ 9 $\underline{7}$ 6 $\underline{4}$ $\underline{1}$	A1	1.1b
		(3)	
(b)	Bin 1: 18 10 1 Bin 2: 17 13 Bin 3: 14 9 7 Bin 4: 6 4	M1 A1	1.1b 1.1b
		(2)	
		(5 marks)	

Notes:

(a)

M1: quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot).

A1: first two passes correct and correct pivots chosen for third pass

A1: cso (correct solution only – all previous marks in this part must have been awarded) – must include a fourth pass

(b)

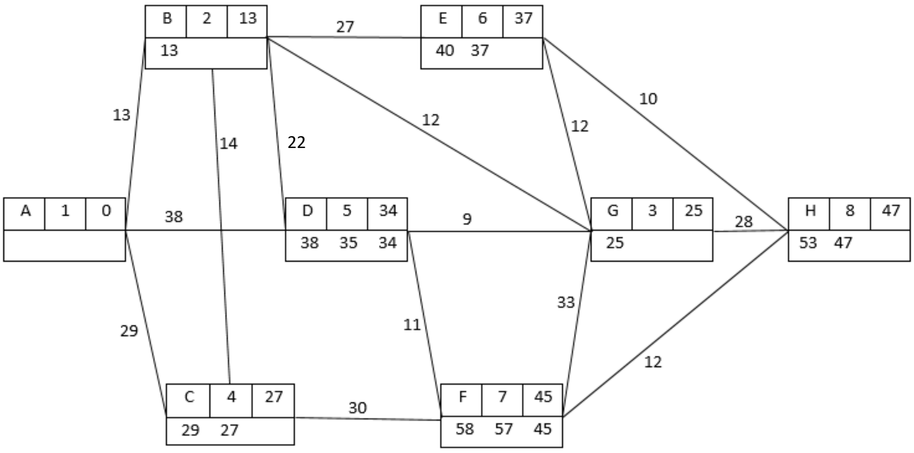
M1: must be using ‘sorted’ list in descending order. First five items placed correctly and at least eight values placed in bins

A1: cso (so no additional/repeated values)

Question	Scheme	Marks	AOs
2(a)	7	B1	2.2a
		(1)	
(b)	A semi-Eulerian graph requires exactly two odd nodes… …the graph has six odd nodes so only two arcs needs to be added to make the graph semi-Eulerian	B1 B1	1.2 2.2a
		(2)	
(c)	Creates two lists of arcs e.g. AB BF BE CE EF EG BG BD Since no arc appears in both lists, the graph is planar (or draws a planar version)	M1 A1 A1	2.1 1.1b 2.4
		(3)	
			(6 marks)

Notes:

(a)
B1: cao
(b)
B1: accurately recalls the fact that a semi-Eulerian graph contains <u>exactly</u> two odd nodes
B1: dependent on previous B mark – cao
(c)
M1: creates two list of arcs (with at least three arcs in each list) which contain no common arcs
A1: cao
A1: correct reasoning that no arc appears in both lists + so the graph is therefore planar

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>3(i)(a)</td><td>


Length of quickest route from A to H is 47 minutes</td><td>M1
A1
A1
A1ft</td><td>1.1b
1.1b
1.1b
2.2a</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(b)</td><td>Shortest path from A to F via H: ABGEHF
Length: 47 + 12 = 59 minutes</td><td>B1
B1ft</td><td>1.1b
2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(c)</td><td>e.g. add 1 to each arc</td><td>M1</td><td>3.5c</td></tr>
<tr><td></td><td>except AB, AD, AC (or EH, GH, FH)</td><td>A1</td><td>2.3</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(ii)(a)</td><td>AB + EH = 13 + 10 = 23*
A(BG)E + B(GE)H = 37 + 34 = 71
A(BGE)H + B(G)E = 47 + 24 = 71
Length of the shortest route is 300 + 23 = 323 km</td><td>M1
A1ft
A1
A1ft</td><td>2.1
1.1b
1.1b
2.2a</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(b)</td><td>Repeat arcs: AB, EH</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4">(13 marks)</td></tr>
</table>

Notes:

(i) (a)

M1: for a larger number replaced by a smaller one in the working values boxes at C, D, E, F or H

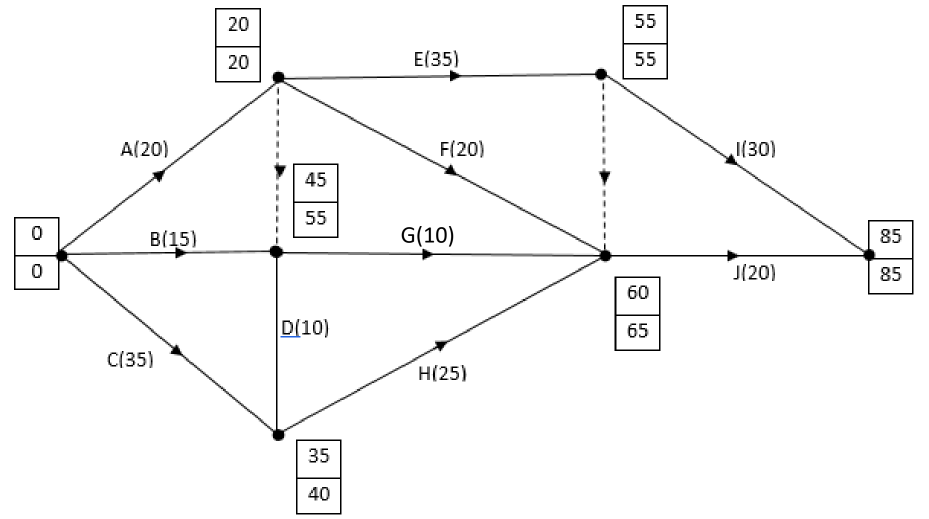
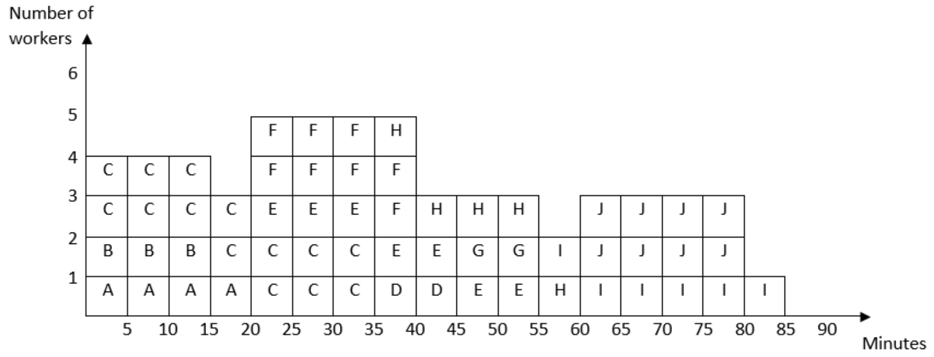
A1: for all values correct (and in correct order) at A, B, G and C

A1: for all values correct (and in correct order) at D, E, F and H

A1ft: for 47 or ft their final value at H

(b)
B1: cao
B1ft: for 59 or ft their final at H + 12
(c)
M1: valid general method – any mention of adding 1 to the weight of the arcs
A1: cao – so adding 1 to each arc except {AB, AD, AC} or {EH, GH, FH}

(ii)(a)
M1: correct three pairings of the required four odd nodes
A1ft: at least two pairings and totals correct (ft their values from (a))
A1: all three pairings and totals correct
A1ft: for 323 or 300 + their shortest repeat
(b)
B1: selecting the shortest pairing, and stating that these arcs should be repeated

Question	Scheme	Marks	AOs
4(a) **(i)**		M1 A1 A1	2.1 1.1b 1.1b
(ii)	Minimum completion time is 85 minutes	A1ft	2.2a
(iii)	Critical activities are A, E and I	A1ft	2.2a
		(5)	
(b)	e.g. 	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(c)	Currently five workers are required between time 20 and 40 and so activities F and H would have to be delayed If F starts at 35 H could not begin until 55 but the latest start time for H is 40. Therefore the project cannot be completed in the minimum time with only four workers	M1 A1	2.4 2.2a
		(2)	
		(10 marks)	

Notes:

(a)(i)

M1: All boxes completed, number generally increasing L to R (condone one "rogue") and decreasing R to L (condone one "rogue")

A1: Cao - Top boxes

A1: Cao - Bottom boxes

(ii)

A1ft: Deduction that result in diagram indicates that project can be completed in 85 minutes

(iii)

A1ft: Deduction of correct critical activities (from their values at each event)

(b)

M1: Plausible histogram with no holes or overhangs (must go to at least 70 on the time axis)

A1: Histogram correct to time 40

A1: Histogram correct from time 40 to time 85

(c)

M1: Explanation involving the need to delay activities F and H

A1: Correct deduction that it is not possible to complete the project with only four workers in the minimum project completion time

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>5(a)</td><td>Maximise $P = 40x + 50y + 65z$
Subject to $\begin{gathered} 3x + 5y + 8z \leq 400 \\ 3x + 6y + 10z \leq 350 \\ x + 1.5y + 1.25z \leq 75 \\ x, y, z \geq 0 \end{gathered}$</td><td>B1
M1
A1
B1</td><td>2.5
3.3
1.1b
3.3</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(b)</td><td>
<table>
<tr><th>b.v.</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>Value</th></tr>
<tr><td>s_1</td><td>3</td><td>5</td><td>8</td><td>1</td><td>0</td><td>0</td><td>400</td></tr>
<tr><td>s_2</td><td>3</td><td>6</td><td>10</td><td>0</td><td>1</td><td>0</td><td>350</td></tr>
<tr><td>s_3</td><td>1</td><td>1.5</td><td>1.25</td><td>0</td><td>0</td><td>1</td><td>75</td></tr>
<tr><td>P</td><td>–40</td><td>–50</td><td>–65</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table>
</td><td>M1
A1</td><td>3.4
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(c)</td><td>
<table>
<tr><th>b.v.</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>Value</th><th>Row Ops</th></tr>
<tr><td>s_1</td><td>0.6</td><td>0.2</td><td>0</td><td>1</td><td>–0.8</td><td>0</td><td>120</td><td>$R_1 = r_1 - 8R_2$</td></tr>
<tr><td>z</td><td>0.3</td><td>0.6</td><td>1</td><td>0</td><td>0.1</td><td>0</td><td>35</td><td>$R_2 = 0.1r_2$</td></tr>
<tr><td>s_3</td><td>0.625</td><td>0.75</td><td>0</td><td>0</td><td>–0.125</td><td>1</td><td>31.25</td><td>$R_3 = r_3 - 1.25R_2$</td></tr>
<tr><td>P</td><td>–20.5</td><td>–11</td><td>0</td><td>0</td><td>6.5</td><td>0</td><td>2275</td><td>$R_4 = r_4 + 65R_2$</td></tr>
</table>
</td><td>M1
A1ft
A1
B1ft</td><td>2.1
1.1b
1.1b
2.4</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(d)</td><td>$P + 13.6y + 2.4s_2 + 32.8s_3 = 3300$ so increasing y, s_2 or s_3 will decrease profit</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(e)</td><td>(i) Make 50 lectern desks, 20 writing desks and no roll top desks</td><td>B1</td><td>3.2a</td></tr>
<tr><td></td><td>(ii) £3300</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(f)</td><td>The 90 is the value of the slack variable s_1 which comes from the constraint $3x + 5y + 8z \leq 400$</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>Indicating that there is 90 m^2 of wood still available</td><td>B1</td><td>3.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
</table>

(g)	e.g. there is no guarantee that all the desks will be sold	B1	3.5b
		(1)	
			(16 marks)

Notes:

(a)

B1: Correct objective function/expression (accept in pence rather than pounds e.g. 4000x + 5000y + 6500z) together with 'maximise'

M1: Correct coefficients and correct right-hand side for at least one inequality – accept any inequality or equals

A1: All three correct (non-trivial) inequalities

B1: $x, y, z \geq 0$

(b)

M1: Constructing all four rows including slack variables with at least one negative in P row (allow sign/numerical slips)

A1: All four rows correct

(c)

M1: Correct pivot located, attempt to divide row

A1ft: Pivot row correct (including change of b.v.) and row operations used at least once, one of columns x, y, s_2 or Value correct

A1: Cao for values (ignore b.v. column and Row Ops)

B1ft: The correct Row Operations (on the ft) explained either in terms of the 'old' or 'new' pivot rows

(d)

B1: States correct objective function and mention of increasing y, s_2 or s_3 will decrease profit

(e)(i)

B1: Cao – in context so not in terms of x, y and z

(ii) B1: Cao

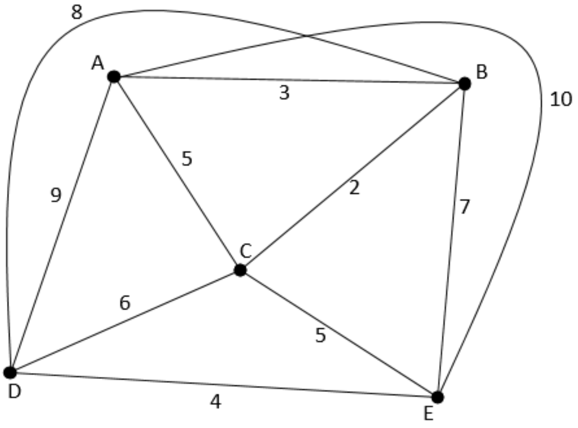
(f)

B1: Recognises that $s_1 = 90$ and is linked to the wood constraint

B1: Evaluates this value in context (so must see both units and mention of 'wood')

(g)

B1: Cao – any suitable limitation to the solution in context

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>6(a)</td><td>
Distance table
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>-</td><td>3</td><td>7</td><td>9</td><td>∞</td></tr>
<tr><td>B</td><td>3</td><td>-</td><td>2</td><td>∞</td><td>8</td></tr>
<tr><td>C</td><td>7</td><td>2</td><td>-</td><td>6</td><td>5</td></tr>
<tr><td>D</td><td>9</td><td>∞</td><td>6</td><td>-</td><td>4</td></tr>
<tr><td>E</td><td>∞</td><td>8</td><td>5</td><td>4</td><td>-</td></tr>
</table>
Route table
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>B</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>C</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>D</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>E</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
</table>
</td><td>B1
B1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>
1st iteration:
Distance table
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>-</td><td>3</td><td>7</td><td>9</td><td>∞</td></tr>
<tr><td>B</td><td>3</td><td>-</td><td>2</td><td>12</td><td>8</td></tr>
<tr><td>C</td><td>7</td><td>2</td><td>-</td><td>6</td><td>5</td></tr>
<tr><td>D</td><td>9</td><td>12</td><td>6</td><td>-</td><td>4</td></tr>
<tr><td>E</td><td>∞</td><td>8</td><td>5</td><td>4</td><td>-</td></tr>
</table>
Route table
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>B</td><td>A</td><td>A</td><td>C</td><td>A</td><td>E</td></tr>
<tr><td>C</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>D</td><td>A</td><td>A</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>E</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
</table>
</td><td>M1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td>
2nd iteration:
Distance table
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>-</td><td>3</td><td>5</td><td>9</td><td>11</td></tr>
<tr><td>B</td><td>3</td><td>-</td><td>2</td><td>12</td><td>8</td></tr>
<tr><td>C</td><td>5</td><td>2</td><td>-</td><td>6</td><td>5</td></tr>
<tr><td>D</td><td>9</td><td>12</td><td>6</td><td>-</td><td>4</td></tr>
<tr><td>E</td><td>11</td><td>8</td><td>5</td><td>4</td><td>-</td></tr>
</table>
Route table
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>A</td><td>B</td><td>B</td><td>D</td><td>B</td></tr>
<tr><td>B</td><td>A</td><td>B</td><td>C</td><td>A</td><td>E</td></tr>
<tr><td>C</td><td>B</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>D</td><td>A</td><td>A</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>E</td><td>B</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
</table>
</td><td>M1
A1ft</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(c)</td><td>


</td><td>M1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(d)(i)
(ii)</td><td>NNA: E – D – C – B – A – E
4 + 6 + 2 + 3 + 10 = 25 km</td><td>B1
B1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
</table>

(e)	E – D – C – B – A – B – C – E	B1	3.2a
		(1)	
(f)	Prim’s algorithm on reduced network starting at B: BC, CE, DE Lower bound = 11 + 3 + 5 = 19 km	B1 B1ft	1.1b 2.2a
		(2)	
(g)	$19 \leq$ optimal ≤ 25	M1 A1	2.2b 1.1b
		(2)	
			(15 marks)

Notes:

(a)
B1: Correct distance table
B1: Correct route table
(b)
M1: No change in the first row and first column of both tables with at least one value in the distance table reduced and one value in the route table changed
A1: cao
M1: No change in the second row and second column of both tables with at least two values in the distance table reduced and two values in the route table changed
A1ft: Correct second iteration follow through from the candidate’s first iteration
(c)
M1: K_5 drawn with at least one shortest distance from the final distance table present
A1: cao
(d)(i)
B1: cao
(ii)
B1: cao
(e)
B1: cao
(f)
B1: correct RMST starting at any node (except A)
B1ft: length of their RMST + 3 + 5
(g)
M1: Their numbers correctly used, accept any inequalities or any indication of interval from their 19 to their 25 (so 19 – 25 can score this mark). Please note that UB > LB for this mark
A1: cao (no follow through on their values) including correct inequalities or equivalent set notation (but condone $19 < \text{optimal} \leq 25$)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>7</td><td>Objective line $\Rightarrow$ e.g. $P - 3x - 4y = 0$</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td>$y \le 10$
$x \ge 4$</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td>Line through (0, 12) and (8, 0) is $y - 12 = -\frac{3}{2}(x - 0)$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>Line through (5, 0) and (10, 10) is $y - 10 = 2(x - 10)$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>$2x - y \le 10 \Rightarrow 2x - y + s_1 = 10$
$y \le 10 \Rightarrow y + s_2 = 10$
$x \ge 4 \Rightarrow x - s_3 + a_1 = 4$
$3x + 2y \ge 24 \Rightarrow 3x + 2y - s_4 + a_2 = 24$</td><td>M1

A1ft
A1</td><td>2.1

1.1b
1.1b</td></tr>
<tr><td></td><td>$a_1 + a_2 = 4 - x + s_3 + 24 + s_4 - 3x - 2y$
$\Rightarrow A = -(a_1 + a_2) = 4x + 2y - s_3 - s_4 - 28$
$\Rightarrow A - 4x - 2y + s_3 + s_4 = -28$</td><td>M1</td><td>2.2a</td></tr>
<tr><td></td><td>e.g.
<table>
<tr><td>b.v.</td><td>x</td><td>y</td><td>s_1</td><td>s_2</td><td>s_3</td><td>s_4</td><td>a_1</td><td>a_2</td><td>Value</td></tr>
<tr><td>s_1</td><td>2</td><td>−1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>10</td></tr>
<tr><td>s_2</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>10</td></tr>
<tr><td>a_1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>−1</td><td>0</td><td>1</td><td>0</td><td>4</td></tr>
<tr><td>a_2</td><td>3</td><td>2</td><td>0</td><td>0</td><td>0</td><td>−1</td><td>0</td><td>1</td><td>24</td></tr>
<tr><td>P</td><td>−3</td><td>−4</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>A</td><td>−4</td><td>−2</td><td>0</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>−28</td></tr>
</table></td><td>M1
A1</td><td>2.1
2.2a</td></tr>
<tr><td colspan="4" align="right">(10 marks)</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">B1: cao for objective function (oe e.g. $P – 3x – 4y = k$)
B1: cao
M1: correct method for finding the equation of the line through (0, 12) and (8, 0)
M1: correct method for finding the equation of the line through (5, 0) and (10, 10)
M1: translate all 4 inequalities into equations – must include all three types of variables (slack, surplus and artificial)
A1ft: two correct equations following their inequalities
A1: all four correct equations
M1: setting up the new objective and substituting for a_1 and a_2
M1: setting up tableau – all six lines with four basic variables
A1: cao (oe e.g. consistent <i>P</i> line with their objective equation)</td></tr>
</table>