9FM0/02: Core Pure Mathematics 02 Mark scheme

Question	Scheme	Marks	AOs
1(a)	$\int \frac{1}{x^2+6x+25}\,dx = \int \frac{1}{(x+3)^2-9+25}\,dx = \int \frac{1}{(x+3)^2+16}\,dx$ or reaches integral in θ if using substitution.	M1	3.1a
	$= k\arctan\left(\frac{x+b}{a}\right)(+c)$ (or $k\theta$ where $4\tan\theta = x + 3$)	M1	1.1b
	$= \frac{1}{4}\arctan\left(\frac{x+3}{4}\right)+c$	A1	1.1b
		(3)	
(b)	$\int_{-3}^{1}\left(1-\frac{25}{x^2+6x+25}\right)dx = \left[x-\frac{25}{4}\arctan\left(\frac{x+3}{4}\right)\right]_{-3}^{1} = (1-...)-(-3-...)$	M1	1.1b
	$= \left(1-\frac{25}{4}\arctan\left(\frac{4}{4}\right)\right)-\left(-3-\frac{25}{4}\arctan 0\right)$	A1ft	1.1b
	$= 4-\frac{25\pi}{16}$	A1	2.1
		(3)	
(c)	Since the graph crosses the x-axis at $x = 0$ the area lies partially below the x-axis, hence the expression does not give the total area as the part below the axis counts as negative which cancels the positive area, so the student is not correct.	B1	2.3
		(1)	
		(7 marks)	

Notes:

(a)

M1: Identifies the need to and completes the square in the numerator to achieve a standard form, or selects the appropriate substitution $x + 3 = 4\tan\theta$. If using substitution, the integrand and dx must be dealt with and an integral in θ reached (or their chosen variable).

M1: Carries out the integration to a form $k\arctan\left(\frac{x+b}{a}\right)$

A1: Correct integral with or without c

(b)

M1: Applies limits to $x - 25\times$"their answer to (a)" and subtracts correct way.

A1ft: A correct unsimplified answer following through their answer to (a).

A1: Correct simplified exact answer.

(c)

B1: As scheme. Must refer to graph crossing the x-axis and signs of areas being different.

Question	Scheme	Marks	AOs
2(a)	A correct method to sum the series, most likely by the method of differences. Look for $\frac{10}{r^2+8r+15}=\frac{A}{r+3}+\frac{B}{r+5}\Rightarrow A=..,B=..$ followed by an attempt at the sum (or with 1 instead of 10). (Induction may be attempted – see alt for (a).)	M1	3.1a
	$\frac{10}{r^2+8r+15}=\frac{5}{r+3}-\frac{5}{r+5}$ or $\frac{1}{r^2+8r+15}=\frac{1/2}{r+3}-\frac{1/2}{r+5}$	B1	1.1b
	$\sum_{r=1}^{n}\frac{10}{r^2+8r+15}=5\sum_{r=1}^{n}\left(\frac{1}{r+3}-\frac{1}{r+5}\right)$ $=5\left[\left(\frac{1}{4}-\cancel{\frac{1}{6}}\right)+\left(\frac{1}{5}-\cancel{\frac{1}{7}}\right)+\left(\cancel{\frac{1}{6}}-\cancel{\frac{1}{8}}\right)+\cdots+\left(\cancel{\frac{1}{n+3}}-\frac{1}{n+5}\right)\right]$	M1	2.1
	$=5\left(\frac{1}{4}+\frac{1}{5}-\frac{1}{n+4}-\frac{1}{n+5}\right)$	A1ft	1.1b
	$=5\left(\frac{5(n+4)(n+5)+4(n+4)(n+5)-20(n+5)-20(n+4)}{20(n+4)(n+5)}\right)=...$	M1	2.1
	$=\frac{9n^2+41n}{4(n+4)(n+5)}$ (So k = 4)	A1	1.1b
		(6)	
(b)	As $n\to\infty$, $T_n\to\frac{9}{4}$ or appropriate investigation tried.	M1	3.4
	Since the sum is increasing towards $\frac{9}{4}$ which is strictly less than 2.5 T_n can never reach 2.5, so the 2.5 million remaining tonnes of coal will not all be mined no matter how long the company keeps mining.	A1	3.2b
		(2)	
(c)	In the first 20 years $T_{20}=\frac{221}{120}$ million tonnes of coal have been mined, so $2.5-\frac{221}{120}=\frac{79}{120}$ tonnes remain.	M1	2.2b
	Hence $\frac{79}{120\times 20}$ extra tonnes per year need mining, so the new model is $M_r=\frac{79}{2400}+\frac{10}{r^2+8r+15}$.	A1ft	3.5c
		(2)	
			(10 marks)

Notes:

(a)
M1: Attempts the sum using an appropriate method – ie method of differences. An attempt at partial fractions would evidence the attempt.
B1: Correct split into partial fractions.

M1: Applies method of differences showing evidence of the cancelling terms. The 5 may be missing at this stage and included later.

A1ft: Correct non-cancelling terms identified. Follow through their split into partial fractions if it leads to most terms cancelling.

M1: Puts the terms over a common denominator and simplifies. May be done in stages with the numerical fractions combined first etc, but look for appropriately adapted numerators for their method.

A1: Correct form with $k = 4$.

(b)

M1: Investigates the long term behaviour, e.g. by trying large values of n in the expression to see what happens, or by considering the long term limit.

A1: As scheme, comments that since the limit of the sum as $n \to \infty$ is 9/4 then the total amount of coal mined will never exceed 2.25 million tonnes, and so the coal will not all be mined even after a long time.

(c)

M1: Calculates the shortfall between 2.5 and the value of the sum at $n = 20$.

A1ft: Correct adaptation of the model adding (their shortfall)/20 to the original expression.

Alt (a)	Use of induction: Look for an attempt to find the value of k using $n = 1$ followed by an attempt at the inductive hypothesis.	M1	3.1a
	$n=1 \Rightarrow \frac{10}{1+8+15} = \frac{9+41}{k(5)(6)} \Rightarrow k=4$	B1	1.1b
	Assume true for $n = p$, so $\sum_{r=1}^{p+1} M_r = \frac{9p^2+41p}{"4"(p+4)(p+5)} + \frac{10}{(p+1)^2+8(p+1)+15}$ $= \frac{9p^2+41p}{"4"(p+4)(p+5)} + \frac{10}{(p+4)(p+6)}$	M1	2.1
	$= \frac{(9p^2+41p)(p+6)+10\times"4"(p+5)}{"4"(p+4)(p+5)(p+6)}$	M1 A1ft	1.1b 1.1b
	$= \frac{9p^3+95p^2+286p+200}{4(p+4)(p+5)(p+6)} = \frac{(p+4)(p+1)(9p+50)}{4(p+4)(p+5)(p+6)}$ $= \frac{(p+1)\left[9(p+1)^2+41\right]}{4((p+1)+4)((p+1)+5)}$ Hence true for $n = 1$ (with $k = 4$) and if true for $n = p$ then true for $n = p + 1$ so true for all positive integers n.	A1	2.1
		(6)	

M1: For use of induction look for an attempt to find the value of k first, followed by an attempt at proving the inductive step.

B1: Deduces $k = 4$.

M1: Assumes true for some p and uses their k in the expression for T_p (may use k instead of p, which is fine if there is no confusion as they have a value in the expression).

M1: Attempts to combine over a common denominator.

A1ft: Correct single fraction expression, follow through their k.

A1: Completes the induction step and make a suitable conclusion.

Question	Scheme	Marks	AOs
3(a)	$\begin{pmatrix}3 & 3\\4 & 7\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}\Rightarrow \begin{matrix}3x+3y=x\\4x+7y=y\end{matrix}$	M1	1.1b
	$\Rightarrow \begin{matrix}2x=-3y\\4x=-6y\end{matrix}\Rightarrow y=-\frac{2}{3}x$	M1	1.1b
	So the invariant points of the transformation are precisely those points lying on the line $y=-\frac{2}{3}x$.	A1	2.4
		(3)	
(b)	$\begin{pmatrix}3 & 3\\4 & 7\end{pmatrix}\begin{pmatrix}x\\mx+c\end{pmatrix}=\begin{pmatrix}x'\\mx'+c\end{pmatrix}\Rightarrow \begin{matrix}3x+3(mx+c)=x'\\4x+7(mx+c)=mx'+c\end{matrix}$	M1	3.1a
	$\Rightarrow 4x+7(mx+c)=m(3x+3mx+3c)+c\Rightarrow(..)x+..=..$	M1	1.1b
	$\Rightarrow(4+7m-3m-3m^2)x+7c=3mc+c$ (oe)	A1	1.1b
	$\Rightarrow(2-m)(2+3m)x+3c(2-m)=0\Rightarrow(2-m)\left((2+3m)x+3c\right)=0$ $\Rightarrow m-2=0$ or both $2+3m=0$ and $c=0$ (since the equation must hold for all x to give fixed lines)	M1	3.1a
	Since the equations are satisfied whenever $m = 2$, the lines $y = 2x+c$ are invariant lines under T.	A1	2.4
	Also, as the equation holds when $m = -2/3$ and $c = 0$, the line $y=-\frac{2}{3}x$ is invariant – or notes that this line is invariant as all the points on it are invariant as shown in (a).	B1	2.2a
		(6)	
		(9 marks)	

Notes:

(a)

M1: Sets up a matrix equation to find the fixed points and extracts a pair of simultaneous equations. (May just see the simultaneous equations.)

M1: Solves the equations showing the same solution comes from both.

A1: Describes the invariant points as those on the line $y=-\frac{2x}{3}$.

(b)

M1: Sets up a matrix equation to find the fixed lines and extracts a pair of simultaneous equations.

M1: Substitutes for x' and expands and gathers terms in x.

A1: Correct equation with terms in x gathered.

M1: Factorises the quadratic in m and factors out the common term – cancelling the term without consideration of it is M0. Since the equation must hold for any x they must deduce this occurs when the factor $m - 2 = 0$ or when $(3m + 2) = 0$ and $c = 0$.

A1: Explains that when $m = 2$ the line is fixed so the lines $y = 2x + c$ for any c are invariant under T...

B1: … and the line $y=-\frac{2x}{3}$ is also invariant since this satisfies $m=-\frac{2}{3}$ and $c = 0$, or since all the points on it are invariant from (a). This mark is not dependent on any others so can be scored if they deduce this line is invariant directly from (a).

Question	Scheme	Marks	AOs
4(a)	$\lvert w-2\rvert^2 = (w-2)(w-2)^* = (w-2)(w^*-2)$	M1	1.1b
	$= ww^* - 2w - 2w^* + 4 = \lvert w\rvert^2 - 2(w + w^*) + 4$	M1	1.1b
	$= 1 + 4 - 2(w + w^*) = 5 - 2(w + w^*)$ since w is a root of unity so has modulus 1. *	A1*	2.1
		(3)	
Alt	$w = x + \mathrm{i}y \Rightarrow \lvert w-2\rvert^2 = \lvert (x-2) + \mathrm{i}y\rvert^2 = (x-2)^2 + y^2$	M1	1.1b
	$= x^2 - 4x + 4 + y^2 = x^2 + y^2 + 4 - 2(x + \mathrm{i}y + x - \mathrm{i}y)$	M1	1.1b
	$= 1 + 4 + 2(w + w^*)$ since $x^2 + y^2 = 1$ as w is a root of unity. *	A1*	2.1
		(3)	
(b)	$\sum_{i=1}^{7}(XA_i)^2 = \sum_{i=1}^{7}\lvert w_i - 2\rvert^2$ where w_i are the 7th roots of unity.	M1	3.1a
	$= \sum_{i=1}^{7}\left(5 - 2(w_i + w_i^*)\right) = \sum_{i=1}^{7}5 - 2\sum_{i=1}^{7}(w_i + w_i^*)$	M1	1.1b
	$\sum_{i=1}^{7}(w_i + w_i^*) = 0$ since roots of unity sum to zero.	B1	2.2a
	So $\sum_{i=1}^{7}(XA_i)^2 = 7 \times 5 = 35$	A1	1.1b
		(4)	
			(7 marks)

Notes:

(a)
M1: Uses the given identity and distributivity of the conjugate.
M1: Expands and collects terms
A1*: Completes the proof with justification of $\lvert w\rvert = 1$.
Alt
M1: Replaces w by $x + \mathrm{i}y$ and applied the modulus squared.
M1: Expands the brackets and gathers $x^2 + y^2$ (may be implied if $x^2 + y^2 = 1$ stated explicitly) and splits the x term (may be implied if $w + w^* = 2x$ stated explicitly).
A1*: Completes proof convincingly with justification for $x^2 + y^2 = 1$ given.

(b)
M1: Makes the connection with part (a) and translates into a complex plane problem, realising the vertices lie at 7th roots of unity.
M1: Uses the identity shown in (a) and splits the sum.
B1: Deduces the second sum is zero as sum of roots of unity is zero.
A1: Correct answer.

Question	Scheme	Marks	AOs
5(a)	$\frac{dy}{dx}=\frac{1}{\sinh^2 x+1}\times..$	M1	1.2
	$\frac{dy}{dx}=\frac{1}{\sinh^2 x+1}\times\cosh x$	A1	1.1b
	$=\frac{\cosh x}{\cosh^2 x}=\text{sech}\,x$ or use of correct identity $\sinh^2 x + 1 = \cosh^2 x$ later in the proof.	B1	2.1
	E.g. $\frac{d^2y}{dx^2}=-\text{sech}\,x\tanh x$ or $\frac{d^2y}{dx^2}=-(\cosh x)^{-2}\times\sinh x$ or even $\frac{d^2y}{dx^2}=\frac{(\sinh x)(\sinh^2 x+1)-(\cosh x)(2\sinh x\cosh x)}{(\sinh^2 x+1)^2}$	M1	1.1b
	$\frac{d^3y}{dx^3}=-(-\text{sech}\,x\tanh x)(\tanh x)+(-\text{sech}\,x)(\text{sech}^2 x)$ (oe) or any valid attempt at the third derivative from their second derivative. E.g. $\frac{d^2y}{dx^2}=-\tanh x\frac{dy}{dx}$ then $\frac{d^3y}{dx^3}=-\text{sech}^2 x\frac{dy}{dx}-\tanh x\frac{d^2y}{dx^2}$	M1 A1	3.1a 1.1b
	E.g. $\frac{d^3y}{dx^3}=\text{sech}\,x\tanh^2 x-\text{sech}^3 x=\text{sech}\,x(1-\text{sech}^2 x)-\text{sech}^3 x$ $=\text{sech}\,x-2\text{sech}^3 x=\frac{dy}{dx}-2\left(\frac{dy}{dx}\right)^3$ * **or** $\frac{d^3y}{dx^3}=-\text{sech}^2 x\frac{dy}{dx}-\tanh x\frac{d^2y}{dx^2}=-\left(\frac{dy}{dx}\right)^3+\tanh^2 x\frac{dy}{dx}$ $=(1-\text{sech}^2 x)\frac{dy}{dx}-\left(\frac{dy}{dx}\right)^3=\frac{dy}{dx}-2\left(\frac{dy}{dx}\right)^3$ *	A1*	2.1
		(7)	
(b)	$\frac{d^4y}{dx^4}=\frac{d^2y}{dx^2}-6\left(\frac{dy}{dx}\right)^2\times\frac{d^2y}{dx^2}$	M1 A1	1.1b 1.1b
	$\frac{d^5y}{dx^5}=\frac{d^3y}{dx^3}-12\left(\frac{dy}{dx}\right)\times\left(\frac{d^2y}{dx^2}\right)^2-6\left(\frac{dy}{dx}\right)^2\frac{d^3y}{dx^3}$	M1 A1	2.1 1.1b
		(4)	
(c)	At $x=0, y=0, y'=1, y''=0, y^{(3)}=-1, y^{(4)}=0$ and $y^{(5)}=-1-1\times0^2-6\times1^2\times(-1)=5$	M1	1.1b
	So $y=y(0)+xy'(0)+\frac{x^2}{2!}y''(0)+\frac{x^3}{3!}y^{(3)}(0)+\frac{x^4}{4!}y^{(4)}(0)+\frac{x^5}{5!}y^{(5)}(0)+...$ with their evaluated values.	M1	1.1b
	$y=x-\frac{x^3}{6}+\frac{x^5}{24}+...$	A1	2.5
		(3)	
			(14 marks)

Notes:
(a) **M1:** Applies correct derivative of arctan(..) **A1:** Correct derivative of *y*. **B1**: Uses the identity $1+\sinh^2 x = \cosh^2 x$ to simplify the expression or anywhere later in their proof. **M1:** Attempts the second derivative either using standard results, or quotient rule on unsimplified form. **M1:** Simplifies and attempts the third derivative or attempts third derivative before simplifying. May even replace sech *x* with *y*' in the second derivative before using product rule. Many routes are possible at this stage (but must use product rule, chain rule, quotient rule as appropriate) **A1**: A correct third derivative in any form. **A1***: Fully correct work leading to the given answer. Steps should be clear to reach the given answer.
(b) **M1:** Differentiates again using the chain rule on the cube term. Constant multiple may be incorrect. **A1:** Correct (unsimplified) fourth derivative. **M1:** Completes the process of differentiation to reach the 5^{th} derivative. **A1:** Correct answer, need not be simplified. Isw after a correct expression.
(c) **M1:** Attempts the evaluation of all the derivatives at $x = 0$. **M1:** Applies the Maclaurin formula with their values. Accept with 3! or 6 and with 5! or 120. **A1:** Correct series, must start $y = ...$ or with $f(x) = ...$ only if this has been defined as being equal to *y* at some stage in their working.

Question	Scheme	Marks	AOs
6(a)	$\frac{d^2x}{dt^2}+6\frac{dx}{dt}+9x=2e^{-3t}$		
	AE: $m^2+6m+9=0 \Rightarrow (m+3)^2=0 \Rightarrow m=\ldots\ (=-3)$	M1	1.1b
	So C.F. is $x_{CF}=(A+Bt)e^{-3t}$	A1	2.2a
	For P.I. try $x_{PI}=kt^2e^{-3t}$	B1	2.2a
	$\dot{x}_{PI}=2kte^{-3t}-3kt^2e^{-3t}\quad (=k(2t-3t^2)e^{-3t})$ $\ddot{x}_{PI}=2ke^{-3t}-6kte^{-3t}-6kte^{-3t}+9kt^2e^{-3t}\quad (=k(2-12t+9t^2)e^{-3t})$ $\Rightarrow k(2-12t+9t^2)e^{-3t}+6k(2t-3t^2)e^{-3t}+9kt^2e^{-3t}=2e^{-3t}\Rightarrow k=\ldots$	M1	1.1b
	So $k=1$ ie $x_{PI}=t^2e^{-3t}$	A1	1.1b
	General solution is $x=(A+Bt)e^{-3t}+t^2e^{-3t}$ (their C.F. + their P.I.)	M1	1.1a
	$x(0)=20 \Rightarrow A=20$	M1	3.4
	$\dot{x}=Be^{-3t}-3(A+Bt)e^{-3t}+2te^{-3t}-3t^2e^{-3t}=(B-3A+(2-3B)t-3t^2)e^{-3t}$ $\dot{x}(0)=100\Rightarrow B=100+3A=\ldots(=160)$	M1	3.4
	So $x=(20+160t+t^2)e^{-3t}$	A1	1.1b
		(9)	
(b)	From above $\dot{x}=(B-3A+(2-3B)t-3t^2)e^{-3t}=(100-478t-3t^2)e^{-3t}$		
	$\dot{x}=0\Rightarrow 100-478t-3t^2=0\Rightarrow t=..\ (=-159.5\ldots$ or $0.2089\ldots)$	M1	3.1a
	$t>0$, so $t_{max}=0.2089\ldots\Rightarrow$ $x_{max}=\left(20+160\times0.2089\ldots+(0.2089...)^2\right)e^{-3\times0.2089\ldots}=\ldots$	M1	3.4
	$x_{max}=$ awrt 28.6 cm (3 s.f.) (28.57055381741878)	A1	1.1b
		(3)	
(c)	$x(2.86)=0.0912\ldots$ which is close to zero (less than 1mm), which can be accounted for by inaccuracies in measurements. So the model is supported by this measurement.	B1ft	2.2b
		(1)	
		(13 marks)	

Notes:

(a)

M1: Forms and solves the auxiliary equation.

A1: Deduces correct C.F. for repeated root. (Variables must be consistent.)

B1: Deduces a correct form for the P.I. following a correct C.F. Accept any variations that include kt^2e^{-3t} with other terms.

M1: Differentiates their P.I. twice and substitutes into original equation and attempts to find the unknown(s).

A1: Correct value for k or correct P.I.

M1: Forms general solution, x = their C.F. + their P.I.

M1: Uses $x=20$ at $t=0$ to find first constant/set up one equation in two unknowns.

M1: Differentiates general solution and uses $\dot{x}=100$ at $t=0$ to form and solve second equation in the unknowns.

A1: Correct answer.

(b) **M1:** Uses $\dot{x} = 0$ to find the time the maximum is achieved. May use the derivative from (a) with constants found, or may differentiate again from answer to (a). **M1:** Substitutes t_{max} into their equations to find x_{max}. **A1:** Correct answer.
(c) **B1ft:** Finds x when $t = 2.86$ and makes an inference about whether it supports the model or not. The conclusion should be relevant for their found value, if close to zero then should conclude in accordance with model as may have slight variance due to measurements not being accurate, if not close to zero, then should conclude that even taking inaccuracies into account the measurement does not fit with the model.

Question	Scheme	Marks	AOs
7(a)	$\int_k^8 \left((4k^2-1)y-(32k^2-k)\right)\,dy = \left[(4k^2-1)\frac{y^2}{2}-(32k^2-k)y\right]_k^8$ $=(4k^2-1)\frac{8^2-k^2}{2}-(32k^2-k)(8-k)$	M1	1.1b
	$=\frac{1}{2}(4k^2-1)(8-k)(8+k)-(32k^2-k)(8-k)$ $=\frac{1}{2}(8-k)\left((4k^2-1)(8+k)-2(32k^2-k)\right)$	M1	3.1a
	$=\frac{1}{2}(8-k)\left(32k^2+4k^3-8-k-64k^2+2k\right)$ $=\frac{1}{2}(8-k)\left(4k^3-32k^2+k-8\right)$*	A1*	2.1
		(3)	
(b)	Uses $(\pi)\int x^2\,dy$ with both of the curves and adds the results (a complete method to find the volume of the main body piece).	B1	3.1a
	Attempts $\int x^2 dy = \int \frac{y^6}{k^4}\,dy = ..$	M1	1.1b
	So $(\pi)\int_0^k x^2\,dy = \frac{(\pi)k^3}{7}$	A1	2.2a
	Attempts second curve $\int x^2 dy = \int \frac{(4k^2-1)y-(32k^2-k)}{-(32-4k)}\,dy = ..$	M1	1.1b
	$(\pi)\int_k^8 x^2 dy = \frac{\frac{1}{2}(8-k)\left(4k^3-32k^2+k-8\right)}{-4(8-k)} = (\pi)\frac{1}{8}\left(8-k+32k^2-4k^3\right)$	M1	2.1
	So volume of body $=(\pi)\left(\frac{k^3}{7}+\frac{1}{8}\left(8-k+32k^2-4k^3\right)\right)$ Or total volume $=(\pi)\left(1+\frac{k^3}{7}+\frac{1}{8}\left(8-k+32k^2-4k^3\right)\right)$	A1	1.1b
	$\frac{dV}{dk}=0 \Rightarrow \frac{3k^2}{7}+\frac{1}{8}\left(1+64k-12k^2\right)=0$	M1	3.1a
	$\Rightarrow 60k^2-448k+7=0 \Rightarrow k=...$	M1	1.1b
	But $k > ½$ so must be k = awrt 7.45 cm	A1	3.2a
		(9)	

(c)	Volume of handle is $\pi r^2 h$ $(= \pi(0.5)^2 \times 4) = \pi$	B1	2.2a
	So volume of spinning top is $V = \pi\left(1 + \frac{(7.45)^3}{7} + \frac{1}{8}\left(8 - (7.45) + 32(7.45)^2 - 4(7.45)^3\right)\right) = ...$	M1	1.1b
	= awrt 237 cm^3 (3 s.f.)	A1	1.1b
		(3)	
			(15 marks)

Notes:

(a)

M1: Correct attempt at integration and applies the limits.

M1: Applies completion of the square or expanding and factorising to obtain the factor $(k - 8)$ and removes this factor.

A1*: Correct completion, expands and collects terms inside the bracket. No errors seen and sufficient steps must be shown.

(b)

B1: Realises the needed to find the volume and attempts the formula at for both curves, adding the result. Note the π is not necessary at all for part (b).

M1: Attempts to make x^2 the subject of the first equation and attempts to integrate it. Power of k may be incorrect.

A1: Correct limits 0 and k applied to deduce the volume in terms of k for this section.

M1: Attempts the integral for top portion of body, make x^2 the subject, including the $32 - 4k$ in the denominator.

M1: Obtains the result using (a) and cancels the $(8 - k)$ term to achieve a cubic in k.

A1: Adds the results of the integrals to give the volume for the whole spinning top, or just the body. If the volume of the cylinder is incorrect, ignore this term for the accuracy – the non-constant terms should all be correct.

M1: Realises the need to differentiate the result and set equal to 0 to obtain the x value at any stationary points. Must be a valid attempt at differentiating.

M1: Solves the quadratic (usual rules).

A1: Correct answer, with second root rejected, or comment why this root gives the maximum.

(c) Allow marks for part (c) for the volume if the work is done in part (b).

B1: Deduces correct volume π for the handle. Cylinder formula or use of integration may be used. Award wherever seen - could be in (b).

M1: Substitutes their value for k into their volume formula. Dependent on the volume formula having been the sum of the three sections and including the factor π - handle must be included. May have been done in (b).

A1: Awrt 237cm^3 NB if this is given in (b) allow this mark unless a different answer is given in (c), in which case count the answer given in (c) as their answer.