TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION (TMUA)

Content Specification

For assessment in October 2024 and January 2025

Structure of the Test

The test will consist of two 75 minute papers, taken one after the other.

Each paper will consist of 20 multiple-choice questions.

Questions across the two papers carry equal weight and there will be no penalty for incorrect answers, so candidates are advised to attempt all questions.

There is no formulae booklet for this test; students are expected to understand and recall all relevant formulae.

Candidates may **not** use calculators.

The details of the papers are as follows:

Paper 1: Applications of Mathematical Knowledge

Time: 75 minutes

Content: 20 multiple-choice questions

Requirements: Section 1 below

This paper will test the candidate's ability to apply their mathematical knowledge in a variety of contexts. Candidates will be expected to know and use the mathematical content set out in Section 1 below.

Paper 2: Mathematical Reasoning

Time: 75 minutes

Content: 20 multiple-choice questions

Requirements: Sections 1 **and** 2 below

This paper will test the candidate's ability to apply their conceptual knowledge to constructing and analysing mathematical arguments. For this paper candidates are expected to be familiar with the contents of Sections 1 and 2 below.

SECTION 1

*This section sets out the **mathematical knowledge** requirement for both papers of the test. The content of Part 1 is almost all covered within the pure mathematics specification of an AS level in mathematics, and the content of Part 2 is almost all covered within a Higher Level GCSE mathematics course.*

There is some duplication of content across Parts 1 and 2.

Candidates are advised to read through these specifications carefully to ensure they are aware of all topics and areas that might be covered in the test.

Part 1

MM1. Algebra and functions

MM1.1 Laws of indices for all rational exponents.

MM1.2 Use and manipulation of surds.

Simplifying expressions that contain surds, including rationalising the denominator.

For example: simplifying $\frac{\sqrt{5}}{3+2\sqrt{5}}$ and $\frac{3}{\sqrt{7}-2\sqrt{3}}$

MM1.3 Quadratic functions and their graphs; the discriminant of a quadratic function; completing the square; solution of quadratic equations.

MM1.4 Simultaneous equations: analytical solution by substitution, e.g. of one linear and one quadratic equation.

MM1.5 Solution of linear and quadratic inequalities.

MM1.6 Algebraic manipulation of polynomials, including:

a. expanding brackets and collecting like terms

b. factorisation and simple algebraic division (by a linear polynomial, including those of the form $ax + b$, and by quadratics, including those of the form $ax^2 + bx + c$)

c. use of the Factor Theorem and the Remainder Theorem

MM1.7 Qualitative understanding that a function is a many-to-one (or sometimes just a one-to-one) mapping.

Familiarity with the properties of common functions, including $f(x)=\sqrt{x}$ (which always means the 'positive square root') and $f(x)=|x|$

MM2. Sequences and series

MM2.1 Sequences, including those given by a formula for the n^{th} term and those generated by a simple recurrence relation of the form $x_{n+1} = f(x_n)$

MM2.2 Arithmetic series, including the formula for the sum of the first n natural numbers.

MM2.3 The sum of a finite geometric series.

The sum to infinity of a convergent geometric series, including the use of $|r| < 1$

MM2.4 Binomial expansion of $(1 + x)^n$ for positive integer n, and for expressions of the form $(a + f(x))^n$ for positive integer n and simple $f(x)$.

The notations $n!$ and $\binom{n}{r}$.

MM3. Coordinate geometry in the (x, y)-plane

MM3.1 Equation of a straight line, including:

a. $y - y_1 = m(x - x_1)$

b. $ax + by + c = 0$

Conditions for two straight lines to be parallel or perpendicular to each other.

Finding equations of straight lines given information in various forms.

MM3.2 Coordinate geometry of the circle, using the equation of a circle in the forms:

a. $(x - a)^2 + (y - b)^2 = r^2$

b. $x^2 + y^2 + cx + dy + e = 0$

MM3.3 Use of the following circle properties:

a. The perpendicular from the centre to a chord bisects the chord.

b. The tangent at any point on a circle is perpendicular to the radius at that point.

c. The angle subtended by an arc at the centre of a circle is twice the angle subtended by the arc at any point on the circumference.

d. The angle in a semicircle is a right angle.

e. Angles in the same segment are equal.

f. The opposite angles in a cyclic quadrilateral add to 180°.

g. The angle between the tangent and chord at the point of contact is equal to the angle in the alternate segment.

MM4. Trigonometry

MM4.1 The sine and cosine rules, and the area of a triangle in the form $\frac{1}{2}ab\sin C$.

The sine rule includes an understanding of the 'ambiguous' case (angle–side–side).

Problems might be set in 2 or 3 dimensions.

MM4.2 Radian measure, including use for arc length and area of sector and segment.

MM4.3 The values of sine, cosine and tangent for the angles: 0°, 30°, 45°, 60°, 90°.

MM4.4 The sine, cosine and tangent functions; their graphs, symmetries, and periodicity.

MM4.5 Knowledge and use of the equations:

a. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

b. $\sin^2\theta + \cos^2\theta = 1$

MM4.6 Solution of simple trigonometric equations in a given interval (this may involve the use of the identities in 4.5).

For example: $\tan x = -\dfrac{1}{\sqrt{3}}$ for $-\pi < x < \pi$ $\qquad$ $\sin^2(2x + \frac{\pi}{3}) = \frac{1}{2}$ for $-2\pi < x < 2\pi$

$12\cos^2 x + 6\sin x - 10 = 2$ for $0° < x < 360°$

MM5. Exponentials and logarithms

MM5.1 $y = a^x$ and its graph, for simple positive values of a.

MM5.2 Laws of logarithms:

a. $a^b = c \Leftrightarrow b = \log_a c$

b. $\log_a x + \log_a y = \log_a(xy)$

c. $\log_a x - \log_a y = \log_a\left(\dfrac{x}{y}\right)$

d. $k\log_a x = \log_a(x^k)$

including the special cases:

e. $\log_a\left(\dfrac{1}{x}\right) = -\log_a x$

f. $\log_a a = 1$

Questions requiring knowledge of the change of base formula will not be set.

MM5.3 The solution of equations of the form $a^x = b$, and equations which can be reduced to this form, including those that need prior algebraic manipulation.

For example: $3^{2x} = 4$ and $25^x - 3\times 5^x + 2 = 0$

MM6. Differentiation

MM6.1 The derivative of $f(x)$ as the gradient of the tangent to the graph $y = f(x)$ at a point.

a. Interpretation of a derivative as a rate of change.

b. Second-order derivatives.

c. Knowledge of notation: $\frac{\mathrm{d}y}{\mathrm{d}x}$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, $f'(x)$, and $f''(x)$

Differentiation from first principles is excluded.

MM6.2 Differentiation of x^n for rational n, and related sums and differences. This might require some simplification before differentiating.

For example, the ability to differentiate an expression such as $\frac{(3x+2)^2}{x^{\frac{1}{2}}}$

MM6.3 Applications of differentiation to gradients, tangents, normals, stationary points (maxima and minima only), strictly increasing functions [if $f'(x) > 0$] and strictly decreasing functions [if $f'(x) < 0$]. Points of inflexion will not be examined, although a qualitative understanding of points of inflexion in the curves of simple polynomial functions is expected.

MM7. Integration

MM7.1 Definite integration as related to the 'area between a curve and an axis'. The difference between finding a definite integral and finding the area between a curve and an axis is expected to be understood.

MM7.2 Finding definite and indefinite integrals of x^n for n rational, $n \neq 1$, and related sums and differences, including expressions which require simplification prior to integrating.

For example: $\int (x+2)^2 \mathrm{d}x$ and $\int \frac{(3x-5)^2}{x^{\frac{1}{2}}} \mathrm{d}x$

MM7.3 An understanding of the Fundamental Theorem of Calculus and its significance to integration. Simple examples of its use may be required in the forms:

a. $\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$, where $F'(x) = f(x)$

b. $\frac{\mathrm{d}}{\mathrm{d}x}\int_a^x f(x)\,\mathrm{d}x = f(x)$

MM7.4 Combining integrals with either equal or contiguous ranges.

For example: $\int_2^5 f(x)\mathrm{d}x + \int_2^5 g(x)\mathrm{d}x = \int_2^5 [f(x)+g(x)]\mathrm{d}x$

$$\int_2^4 f(x)\mathrm{d}x + \int_4^3 f(x)\mathrm{d}x = \int_2^3 f(x)\mathrm{d}x$$

MM7.5 Approximation of the area under a curve using the trapezium rule; determination of whether this constitutes an overestimate or an underestimate.

MM7.6 Solving differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)$

MM8. Graphs of functions

MM8.1 Recognise and be able to sketch the graphs of common functions that appear in this specification: these include lines, quadratics, cubics, trigonometric functions, logarithmic functions, exponential functions, square roots, and the modulus function.

MM8.2 Knowledge of the effect of simple transformations on the graph of $y = f(x)$ with positive or negative value of a as represented by:

a. $y = af(x)$

b. $y = f(x) + a$

c. $y = f(x + a)$

d. $y = f(ax)$

Compositions of these transformations. Knowledge and use of the notation $f(g(x))$.

MM8.3 Understand how altering the values of m and c affects the graph of $y = mx + c$

MM8.4 Understand how altering the values of a, b and c in $y = a(x + b)^2 + c$ affects the corresponding graph.

MM8.5 Use differentiation to help determine the shape of the graph of a given function, including:

a. finding stationary points (excluding inflexions)

b. when the graph is increasing or decreasing

MM8.6 Use algebraic techniques to determine where the graph of a function intersects the coordinate axes; appreciate the possible numbers of real roots that a general polynomial can possess.

MM8.7 Geometric interpretation of algebraic solutions of equations; relationship between the intersections of two graphs and the solutions of the corresponding simultaneous equations.

Part 2

M1. Units

M1.1 Use standard units of mass, length, time, money and other measures.

Use compound units such as speed, rates of pay, unit pricing, density and pressure, including using decimal quantities where appropriate.

M1.2 Change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts.

M2. Number

M2.1 Order positive and negative integers, decimals and fractions.

Understand and use the symbols: $=$, $\neq$, $<$, $>$, $\leq$, $\geq$.

M2.2 Apply the four operations (addition, subtraction, multiplication and division) to integers, decimals, simple fractions (proper and improper) and mixed numbers – any of which could be positive and negative.

Understand and use place value.

M2.3 Use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, and prime factorisation (including use of product notation and the unique factorisation theorem).

M2.4 Recognise and use relationships between operations, including inverse operations.

Use cancellation to simplify calculations and expressions.

Understand and use the convention for priority of operations, including brackets, powers, roots and reciprocals.

M2.5 Apply systematic listing strategies. (For instance, if there are m ways of doing one task and for each of these tasks there are n ways of doing another task, then the total number of ways the two tasks can be done in order is $m \times n$ ways.)

M2.6 Use and understand the terms: *square*, *positive* and *negative square root*, *cube* and *cube root*.

M2.7 Use index laws to simplify numerical expressions, and for multiplication and division of integer, fractional and negative powers.

M2.8 Interpret, order and calculate with numbers written in standard index form (standard form); numbers are written in standard form as $a \times 10^n$, where $1 \leq a < 10$ and n is an integer.

M2.9 Convert between terminating decimals, percentages and fractions.

Convert between recurring decimals and their corresponding fractions.

M2.10 Use fractions, decimals and percentages interchangeably in calculations.

Understand equivalent fractions.

M2.11 Calculate exactly with fractions, surds and multiples of π.

Simplify surd expressions involving squares, e.g. $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$, and rationalise denominators; for example, candidates could be asked to rationalise expressions such as: $\frac{3}{\sqrt{7}}$, $\frac{5}{3+2\sqrt{5}}$, $\frac{7}{2-\sqrt{3}}$, $\frac{3}{\sqrt{5}-\sqrt{2}}$

M2.12 Calculate with upper and lower bounds, and use in contextual problems.

M2.13 Round numbers and measures to an appropriate degree of accuracy, e.g. to a specified number of decimal places or significant figures.

Use inequality notation to specify simple error intervals due to truncation or rounding.

M2.14 Use approximation to produce estimates of calculations, including expressions involving π or surds.

M3. Ratio and proportion

M3.1 Understand and use scale factors, scale diagrams and maps.

M3.2 Express a quantity as a fraction of another, where the fraction is less than 1 or greater than 1.

M3.3 Understand and use ratio notation.

M3.4 Divide a given quantity into two (or more) parts in a given *part* **:** *part* ratio.

Express the division of a quantity into two parts as a ratio.

M3.5 Apply ratio to real contexts and problems, such as those involving conversion, comparison, scaling, mixing and concentrations.

Express a multiplicative relationship between two quantities as a ratio or a fraction.

M3.6 Understand and use proportion.

Relate ratios to fractions and to linear functions.

M3.7 Identify and work with fractions in ratio problems.

M3.8 Define percentage as ‘number of parts per hundred’.

Interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively.

Express one quantity as a percentage of another.

Compare two quantities using percentages.

Work with percentages greater than 100%.

Solve problems involving percentage change, including percentage increase/decrease, original value problems and simple interest calculations.

M3.9 Understand and use direct and inverse proportion, including algebraic representations.

Recognise and interpret graphs that illustrate direct and inverse proportion.

Set up, use and interpret equations to solve problems involving direct and inverse proportion (including questions involving integer and fractional powers).

Understand that x is inversely proportional to y is equivalent to x is proportional to $\frac{1}{y}$.

M3.10 Compare lengths, areas and volumes using ratio notation.

Understand and make links to similarity (including trigonometric ratios) and scale factors.

M3.11 Set up, solve and interpret the answers in growth and decay problems, including compound interest, and work with general iterative processes.

M4. Algebra

M4.1 Understand, use and interpret algebraic notation; for instance: ab in place of $a \times b$; $3y$ in place of $y+y+y$ and $3 \times y$; a^2 in place of $a \times a$; a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$; $\frac{a}{b}$ in place of $a \div b$.

M4.2 Use index laws in algebra for multiplication and division of integer, fractional, and negative powers.

M4.3 Substitute numerical values into formulae and expressions, including scientific formulae.

Understand and use the concepts and vocabulary: *expressions*, *equations*, *formulae*, *identities*, *inequalities*, *terms* and *factors*.

M4.4 Collect like terms, multiply a single term over a bracket, take out common factors, and expand products of two or more binomials.

M4.5 Factorise quadratic expressions of the form x^2+bx+c, including the difference of two squares.

Factorise quadratic expressions of the form ax^2+bx+c, including the difference of two squares.

M4.6 Simplify expressions involving sums, products and powers, including the laws of indices.

Simplify rational expressions by cancelling, or factorising and cancelling.

Use the four rules on algebraic rational expressions.

M4.7 Rearrange formulae to change the subject.

M4.8 Understand the difference between an equation and an identity.

Argue mathematically to show that algebraic expressions are equivalent.

M4.9 Work with coordinates in all four quadrants.

M4.10 Identify and interpret gradients and intercepts of linear functions ($y = mx + c$) graphically and algebraically.

Identify pairs of parallel lines and identify pairs of perpendicular lines, including the relationships between gradients.

Find the equation of the line through two given points, or through one point with a given gradient.

M4.11 Identify and interpret roots, intercepts and turning points of quadratic functions graphically.

Deduce roots algebraically, and turning points by completing the square.

M4.12 Recognise, sketch and interpret graphs of:

a. linear functions
b. quadratic functions
c. simple cubic functions
d. the reciprocal function: $y = \frac{1}{x}$ with $x \neq 0$
e. the exponential function: $y = k^x$ for positive values of k
f. trigonometric functions (with arguments in degrees): $y = \sin x,\ y = \cos x,\ y = \tan x$ for angles of any size

M4.13 Interpret graphs (including reciprocal graphs and exponential graphs) and graphs of non-standard functions in real contexts to find approximate solutions to problems, such as simple kinematic problems involving distance, speed and acceleration.

M4.14 Calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance–time graphs, speed–time graphs and graphs in financial contexts.

M4.15 Set up and solve, both algebraically and graphically, simple equations including simultaneous equations involving two unknowns; this may include one linear and one quadratic equation.

Solve two simultaneous equations in two variables (linear/linear or linear/quadratic) algebraically.

Find approximate solutions using a graph.

Translate simple situations or procedures into algebraic expressions or formulae; for example, derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution.

M4.16 Solve quadratic equations (including those that require rearrangement) algebraically by factorising, by completing the square, and by using the quadratic formula.

Know the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Find approximate solutions of quadratic equations using a graph.

M4.17 Solve linear inequalities in one or two variables.

Represent the solution set on a number line, or on a graph, or in words.

M4.18 Generate terms of a sequence using term-to-term or position-to-term rules.

M4.19 Deduce expressions to calculate the n^{th} term of linear or quadratic sequences.

M5. Geometry

M5.1 Use conventional terms and notation: points, lines, line segments, vertices, edges, planes, parallel lines, perpendicular lines, right angles, subtended angles, polygons, regular polygons and polygons with reflection and/or rotational symmetries.

M5.2 Recall and use the properties of angles at a point, angles on a straight line, perpendicular lines and opposite angles at a vertex.

Understand and use the angle properties of parallel lines, intersecting lines, triangles and quadrilaterals.

Calculate and use the sum of the interior angles, and the sum of the exterior angles, of polygons.

M5.3 Derive and apply the properties and definitions of special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus.

Derive and apply the properties and definitions of various types of triangle and other plane figures using appropriate language.

M5.4 Understand and use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS).

M5.5 Apply angle facts, triangle congruence, similarity, and properties of quadrilaterals to results about angles and sides.

M5.6 Identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional and negative scale factors).

Describe the changes and invariance achieved by combinations of rotations, reflections and translations.

Describe translations as 2-dimensional vectors.

M5.7 Know and use the formula for Pythagoras' theorem: $a^2 + b^2 = c^2$

Use Pythagoras' theorem in both 2 and 3 dimensions.

M5.8 Identify and use conventional circle terms: *centre*, *radius*, *chord*, *diameter*, *circumference*, *tangent*, *arc*, *sector* and *segment* (including the use of the terms *minor* and *major* for arcs, sectors and segments).

M5.9 Apply the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results:

a. angle subtended at the centre is twice the angle subtended at the circumference
b. angle in a semicircle is 90°
c. angles in the same segment are equal
d. angle between a tangent and a chord (alternate segment theorem)
e. angle between a radius and a tangent is 90°
f. properties of cyclic quadrilaterals

M5.10 Solve geometrical problems on 2-dimensional coordinate axes.

M5.11 Know the terminology *faces*, *surfaces*, *edges* and *vertices* when applied to cubes, cuboids, prisms, cylinders, pyramids, cones, spheres and hemispheres.

M5.12 Interpret plans and elevations of 3-dimensional shapes.

M5.13 Use and interpret maps and scale drawings.

Understand and use three-figure bearings.

M5.14 Know and apply formulae to calculate:

a. the area of triangles, parallelograms, trapezia
b. the volume of cuboids and other right prisms.

M5.15 Know the formulae:

a. circumference of a circle = $2\pi r = \pi d$
b. area of a circle = πr^2
c. volume of a right circular cylinder = $\pi r^2 h$

Formulae relating to spheres, pyramids and cones will be given if needed.

Use formulae to calculate:

a. perimeters of 2-dimensional shapes, including circles
b. areas of circles and composite shapes
c. surface area and volume of spheres, pyramids, cones and composite solids

M5.16 Calculate arc lengths, angles and areas of sectors of circles.

M5.17 Apply the concepts of congruence and similarity in simple figures, including the relationships between lengths, areas and volumes.

M5.18 Know and use the trigonometric ratios:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Apply these to find angles and lengths in right-angled triangles and, where possible, general triangles in 2- and 3-dimensional figures.

Know the exact values of $\sin\theta$ and $\cos\theta$ for $\theta = 0°, 30°, 45°, 60°, 90°$.

Know the exact values of $\tan\theta$ for $\theta = 0°, 30°, 45°, 60°$.

Candidates are not expected to recall or use the sine or cosine rules.

M5.19 Apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors.

Use vectors to construct geometric arguments and proofs.

M6. Statistics

M6.1 Interpret and construct tables, charts and diagrams, including:

a. two-way tables, frequency tables, bar charts, pie charts and pictograms for categorical data

b. vertical line charts for ungrouped discrete numerical data

c. tables and line graphs for time series data

Know the appropriate use of each of these representations.

M6.2 Interpret and construct diagrams for grouped discrete data and continuous data:

a. histograms with equal and unequal class intervals

b. cumulative frequency graphs

Know the appropriate use of each of these diagrams.

Understand and use the term *frequency density*.

M6.3 Calculate the *mean*, *mode*, *median* and *range* for ungrouped data.

Find the modal class; calculate estimates of the range, mean and median for grouped data, and understand why these are estimates.

Describe a population using statistics.

Make simple comparisons.

Compare data sets using like-for-like summary values.

Understand the advantages and disadvantages of summary values.

Calculate estimates of mean, median, mode, range, quartiles and interquartile range from graphical representation of grouped data.

Use the median and interquartile range to compare distributions.

M6.4 Use and interpret scatter graphs of bivariate data.

Recognise correlation, and know that it does not indicate causation.

Draw estimated lines of best fit.

Interpolate and extrapolate apparent trends whilst knowing the dangers of so doing.

M7. Probability

M7.1 Analyse the frequency of outcomes of probability experiments using tables and frequency trees.

M7.2 Apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments.

Understand that if an experiment is repeated, the outcome may be different.

M7.3 Relate relative expected frequencies to theoretical probability, using appropriate language and the ‘0 to 1’ probability scale.

M7.4 Apply the property that the probabilities of an exhaustive set of outcomes sum to one.

Apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one.

M7.5 Enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams. Candidates are not expected to know formal set theory notation.

M7.6 Construct theoretical possibility spaces for single and combined experiments with equally likely outcomes, and use these to calculate theoretical probabilities.

M7.7 Know when to add or multiply two probabilities, and understand conditional probability.

Calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams.

Understand the use of tree diagrams to represent outcomes of combined events:

a. when the probabilities are independent of the previous outcome
b. when the probabilities are dependent on the previous outcome.

SECTION 2

*This section sets out the scope of Paper 2. Paper 2 tests the candidate's ability to think mathematically: the paper will focus on testing the candidate's ability to understand, and construct, mathematical arguments in a variety of contexts. It will draw on the mathematical knowledge outlined in **SECTION 1** above.*

The Logic of Arguments

Arg1 Understand and be able to use mathematical logic in simple situations:

- The terms **true** and **false**;
- The terms **and**, **or** (meaning **inclusive or**), **not**;
- Statements of the form:

  if A then B
  A if B
  A only if B
  A if and only if B

- The **converse** of a statement;
- The **contrapositive** of a statement;
- The relationship between the truth of a statement and its converse and its contrapositive.

Note: candidates will **not** be expected to recognise or use symbolic notation for any of these terms, nor will they be expected to complete formal truth tables.

Arg2 Understand and use the terms **necessary** and **sufficient**.

Arg3 Understand and use the terms **for all**, **for some** (meaning **for at least one**), and **there exists**.

Arg4 Be able to negate statements that use any of the above terms.

Mathematical Proof

Prf1 Follow a proof of the following types, and in simple cases know how to construct such a proof:

- Direct deductive proof ('Since A, therefore B, therefore C, …, therefore Z, which is what we wanted to prove.');
- Proof by cases (for example, by considering even and odd cases separately);
- Proof by contradiction;
- Disproof by counterexample.

Prf2 Deduce implications from given statements.

Prf3 Make conjectures based on small cases, and then justify these conjectures.

Prf4 Rearrange a sequence of statements into the correct order to give a proof for a statement.

Prf5 Problems requiring a sophisticated chain of reasoning to solve.

Identifying Errors in Proofs

Err1 Identifying errors in purported proofs.

Err2 Be aware of common mathematical errors in purported proofs; for example, claiming 'if $ab = ac$, then $b = c$' or assuming 'if $\sin A = \sin B$, then $A = B$' neither of which are valid deductions.